

Matrix Gauss Elimination

page 45. 1.3.

$$2. \quad x + 3y = 2$$

$$3x + 4y = 1$$

Write these equations in the form of a matrix

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & 4 & 1 \end{array} \right]$$

↑
coefficients
of x, y

↑
Right hand side
(RHS) of the
equation

Goal is to make the elements in the diagonal '1'
and elements below the '1' as '0's

Since the first element in the diagonal is '1' we
proceed to make the element below the '1' as '0'

so

$$R_2 + (-3R_1)$$

$$R_1 \rightarrow \begin{array}{ccc|c} 1 & 3 & 2 \end{array}$$

$$-3R_1 \rightarrow \begin{array}{ccc|c} -3 & -9 & -6 \end{array}$$

$$+ R_2 \rightarrow \begin{array}{ccc|c} 3 & 4 & 1 \\ \hline 0 & -5 & -5 \end{array}$$

so

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & -5 & -5 \end{array} \right]$$

Make $-5 \rightarrow '1'$

$$\therefore \left(-\frac{1}{5}\right) \cdot R_2$$

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 1 & 1 \end{array} \right]$$

So Now we have the following equations based on the augmented matrix above

$$x + 3y = 2$$

$$y = 1$$

use $y=1$ in the 1st Eqn

$$x + 3(1) = 2 \quad \therefore x = -1$$

$$\text{Hence } \left. \begin{array}{l} x = -1 \\ y = 1 \end{array} \right\} \text{ soln.}$$

$$12. \quad 2x - 4y + 2z = -6$$

$$3x + 4y + 5z = 1$$

$$2x - y + z = -3$$

$$\left[\begin{array}{ccc|c} 2 & -4 & 2 & -6 \\ 3 & 4 & 5 & 1 \\ 2 & -1 & 1 & -3 \end{array} \right]$$

$$\frac{1}{2} R_1 \rightarrow R_1 \quad \left[\begin{array}{ccc|c} \textcircled{1} & -2 & 1 & -6/2 \\ 3 & 4 & 5 & 1 \\ 2 & -1 & 1 & -3 \end{array} \right]$$

$$R_2 + (-3R_1)$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 10 & 2 & 10 \\ 2 & -1 & 1 & -3 \end{array} \right]$$

$$R_3 + (-2R_1)$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 10 & 2 & 10 \\ 0 & 3 & -1 & 3 \end{array} \right]$$

$$\frac{1}{10} R_2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & \textcircled{1} & 1/5 & 1 \\ 0 & 3 & -1 & 3 \end{array} \right]$$

$$R_3 + (-3R_2)$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1/5 & 1 \\ 0 & 0 & -6/5 & 0 \end{array} \right]$$

$$-\frac{5}{6} R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 0 & 1 & 1/5 & 1 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

③

$$x - 2y + z = -3$$

$$y + \frac{z}{5} = 1$$

$$z = 0$$

$$y = 1$$

$$x - 2(1) + 0 = -3$$

$$x = -3 + 2 = -1$$

$$x = -1$$

$$y = 1$$

$$z = 0$$

16.

$$x + 2y + z = 6$$

$$-x + 3y - 2z = -4$$

$$2x - y - 3z = -8$$

$$\left[\begin{array}{ccc|c} \textcircled{1} & 2 & 1 & 6 \\ -1 & 3 & -2 & -4 \\ 2 & -1 & -3 & -8 \end{array} \right]$$

 $R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & 5 & -1 & 2 \\ 2 & -1 & -3 & -8 \end{array} \right]$$

 $R_3 + (-2R_1)$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & 5 & -1 & 2 \\ 0 & -5 & -5 & -20 \end{array} \right]$$

 $\frac{1}{5}R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & \textcircled{1} & -1/5 & 2/5 \\ 0 & -5 & -5 & -20 \end{array} \right]$$

 $R_3 + (5 \cdot R_2)$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & 1 & -1/5 & 2/5 \\ 0 & 0 & -6 & -18 \end{array} \right]$$

④

 $-\frac{1}{6}R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 6 \\ 0 & 1 & -1/5 & 2/5 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$x + 2y + z = 6$$

$$y - \frac{1}{5}z = \frac{2}{5}$$

$$z = 3$$

$$y - \frac{1}{5}(3) = \frac{2}{5}$$

$$y = \frac{2}{5} + \frac{3}{5} = \frac{5}{5} = 1$$

$$x + 2(1) + 3 = 6$$

$$x = 1$$

$$y = 1$$

$$z = 3$$

Matrix 2.3. INVERSE OF A SQUARE MATRIX.

(5)

Gauss-Jordan Elimination.

$$23 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 2 & 1 & 1 \end{bmatrix} = A$$

Augmented form

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 4 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

A

I

so we want $[A|I]$ to
reduce to $[I|B]$

using row operations

$$\left[\begin{array}{ccc|ccc} \textcircled{1} & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 4 & 0 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

$R_2 - R_1$

$$\left[\begin{array}{ccc|ccc} \textcircled{1} & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 2 & 1 & 1 & 0 & 0 & 1 \end{array} \right]$$

$R_3 + (-2R_1)$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & \textcircled{1} & 3 & -1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array} \right]$$

$R_1 + (-R_2)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 2 & -1 & 0 \\ 0 & \textcircled{1} & 3 & -1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array} \right]$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 2 & -1 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 2 & -3 & 1 & 1 \end{array} \right]$$

$\frac{1}{2}R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 2 & -1 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & \textcircled{1} & -3/2 & 1/2 & 1/2 \end{array} \right]$$

$R_2 + (-3R_3)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 2 & -1 & 0 \\ 0 & 1 & 0 & 7/2 & -1/2 & -3/2 \\ 0 & 0 & \textcircled{1} & -3/2 & 1/2 & 1/2 \end{array} \right]$$

$R_1 + (2R_3)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 7/2 & -1/2 & -3/2 \\ 0 & 0 & 1 & -3/2 & 1/2 & 1/2 \end{array} \right]$$

$$A^{-1} = B = \begin{bmatrix} -1 & 0 & 1 \\ 7/2 & -1/2 & -3/2 \\ -3/2 & 1/2 & 1/2 \end{bmatrix}$$

(6)

$$38 \quad 2x + 4y + 4z = 1$$

$$x + 2y + z = 1$$

$$x - z = 3$$

First write these in matrix form

$$\begin{bmatrix} 2 & 4 & 4 \\ 1 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

A X B

$$AX = B$$

$$\therefore X = A^{-1} \cdot B$$

So we need A^{-1}

$$\left[\begin{array}{ccc|ccc} 2 & 4 & 4 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$\frac{1}{2}R_1$$

$$\left[\begin{array}{ccc|ccc} \textcircled{1} & 2 & 2 & 1/2 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array} \right]$$

$$R_2 - R_1 ; R_3 - R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1/2 & 0 & 0 \\ 0 & 0 & -1 & -1/2 & 1 & 0 \\ 0 & -2 & -3 & -1/2 & 0 & 1 \end{array} \right]$$

observe that the middle diagonal element is zero $\therefore$ swap the Row 2 & Row 3

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1/2 & 0 & 0 \\ 0 & -2 & -3 & -1/2 & 0 & 1 \\ 0 & 0 & -1 & -1/2 & 1 & 0 \end{array} \right]$$

$$\text{Now } \frac{R_2}{-2}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1/2 & 0 & 0 \\ 0 & \textcircled{1} & +3/2 & +1/4 & 0 & -1/2 \\ 0 & 0 & -1 & -1/2 & 1 & 0 \end{array} \right]$$

$$R_1 + (-2R_2)$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 3/2 & 1/4 & 0 & -1/2 \\ 0 & 0 & -1 & -1/2 & 1 & 0 \end{array} \right]$$

$$-1 \cdot R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 3/2 & 1/4 & 0 & -1/2 \\ 0 & 0 & 1 & 1/2 & -1 & 0 \end{array} \right]$$

$$R_1 + R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & 1 \\ 0 & 1 & 3/2 & 1/4 & 0 & -1/2 \\ 0 & 0 & 1 & 1/2 & -1 & 0 \end{array} \right]$$

$$R_2 + (-\frac{3}{2}R_3)$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1 & 1 \\ 0 & 1 & 0 & -1/2 & 3/2 & -1/2 \\ 0 & 0 & 1 & 1/2 & -1 & 0 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 1/2 & -1 & 1 \\ -1/2 & 3/2 & -1/2 \\ 1/2 & -1 & 0 \end{bmatrix}$$

We are not done yet

$$X = A^{-1} B$$

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$$X = \begin{bmatrix} 1/2 & -1 & 1 \\ -1/2 & 3/2 & -1/2 \\ 1/2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{2}(1) + (-1)(1) + 1(3) \\ -\frac{1}{2}(1) + \frac{3}{2}(1) + (-\frac{1}{2})(3) \\ \frac{1}{2}(1) - 1(1) + 0(3) \end{bmatrix}$$

$$X = \begin{bmatrix} 5/2 \\ -1/2 \\ -1/2 \end{bmatrix}$$

$$\left. \begin{array}{l} x = 5/2 \\ y = -1/2 \\ z = -1/2 \end{array} \right\} \text{solution}$$