

Solutions to Suggested Problems

-Exam 1

Note: These are available to help you study. There may be errors or typos. Please see your instructor w/ any questions.

Problems 1-4

1) $x = \# \text{ crates of A}$ $y = \# \text{ crates of B}$

$$M = 15x + 10y$$

A) is not a constraint

2)	T1 A(x)	T2 B(y)	Available
V	5	4	325
W	150	250	12000
P	15	10	

3) $S = D$

$$-.0003q + 14 = .0007q + 4$$

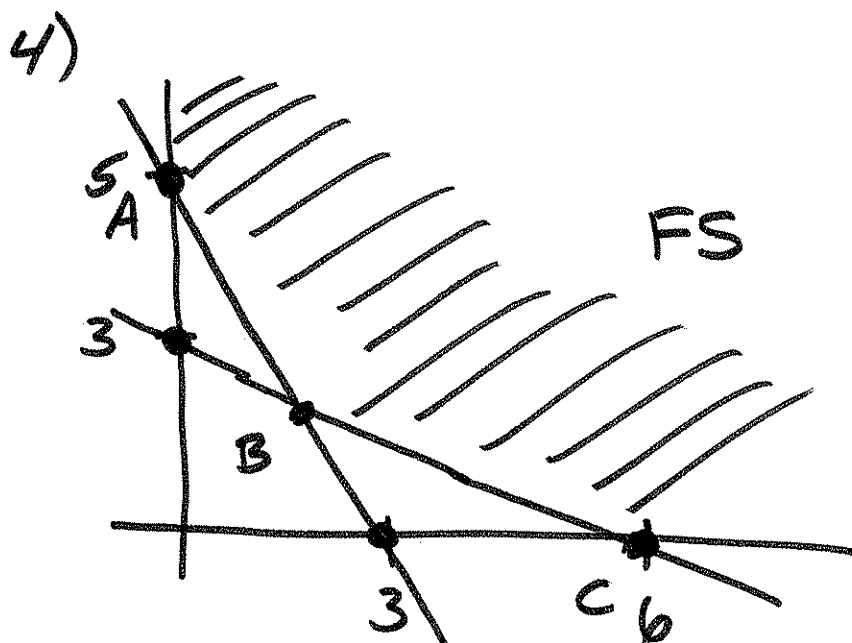
$$10 = .001q$$

$$10000 = q$$

$$S(10,000) = .0007(10,000) + 4 = 11$$

so 10,000 eq quantity

// eq price



A is $(0, 5)$

C is $(6, 0)$

B

$$5x + 3y = 15$$

$$x + 2y = 6 \rightarrow x = 6 - 2y$$

so $5(6 - 2y) + 3y = 15$

$$30 - 10y + 3y = 15$$

$$-7y = -15$$

$$y = 15/7$$

so B is $(12/7, 15/7)$

$$x = 6 - 2(15/7)$$

$$= 6 - 30/7$$

$$= \frac{42 - 30}{7}$$

$$= 12/7$$

$$1.1/ 12, 14, 18, 20, 26, 40, 44$$

$$12) \quad x = \# \text{ units}, \quad p = \text{price} \quad (x, p)$$

$$(1000, 3) \quad (2000, 4)$$

$$m = \frac{4-3}{2000-1000} = \frac{1}{1000}$$

$$p - 4 = \frac{1}{1000} (x - 2000)$$
$$= \frac{1}{1000} x - 2$$

$$p(x) = \frac{1}{1000} x + 2 \quad (\text{Supply})$$

$$14) \quad x = \# \text{ units}, \quad p = \text{price} \quad (x, p)$$

$$(90, 800) \quad (70, 750)$$

$$m = \frac{800-750}{90-70} = \frac{50}{20} = \frac{5}{2}$$

$$p - 800 = \frac{5}{2} (x - 90)$$
$$= \frac{5}{2} x - 225$$

$$p(x) = \frac{5}{2} x + 575 \quad (\text{supply})$$

$$*20) \quad C(x) = \text{variable} + \text{fixed}$$

$$C(x) = .40x + 75 \quad \text{where } x = \# \text{ of miles}$$

*18) $t = \text{time}$ $p = \text{price}$ (t, p)

$(0, 32,000)$ $(8, 23,200)$

$$M = \frac{32000 - 23,200}{0 - 8} = \frac{8800}{-8} = -1100$$

$(0, 32,000)$ is y-intercept

$$p(t) = -1100t + 32,000$$

26) $x = \# \text{ ordered}$ $c = \text{cost}$ (x, c)

$(75, 67.5)$ $(200, 160)$

↓
 $.9 \times 75$

↓
 $.8 \times 200$

$$M = \frac{160 - 67.5}{200 - 75} = \frac{92.5}{125} = .74$$

$$c - 67.5 = .74(x - 75)$$

$$= .74x - 55.5$$

$$C(x) = .74x + 12 \quad (\text{cost})$$

$$\begin{aligned} 40) \quad (x, c) &= (8, .157) \\ &= (10, .190) \end{aligned}$$

$$M = \frac{.190 - .157}{10 - 8} = .0165$$

$$\begin{aligned} C - .157 &= .0165(x - 8) \\ &= .0165x - .132 \end{aligned}$$

$$C(x) = .0165x + .025 \quad (\text{cost})$$

$$44) \quad x = \# \text{ of acres}$$

$$6 \text{ tons/acre} \Rightarrow 6x = \# \text{ of tons}$$

$$\text{at } \$617/\text{ton} \Rightarrow 617(6x) = \# \text{ of dollars}$$

$$R(x) = 3702x$$

1.2/ 4, 6, 10, 16, 17, 28

4) $C = R$

$$.03x + 1 = .04x$$

$$1 = .01x$$

$$100 = x$$

(break even quantity)

6) $S = D$

$$-3x + 12 = 2x + 5$$

$$7 = 5x$$

$$x = 7/5$$

$$P = -3(7/5) + 12 = 39/5$$

eg. pt $(7/5, 39/5)$

10) $x = \# \text{ of movers}$

$$C(x) = 200x + 1,000,000$$

$$R(x) = 300x$$

$$P(x) = R(x) - C(x)$$

$$= 300x - (200x + 1,000,000)$$

$$= 100x - 1,000,000$$

Break-even $C = R$

$$200x + 1,000,000 = 300x$$

$$1,000,000 = 100x$$

$$10,000 = x$$

(break-even quantity)

16) $x = \# \text{ of hrs}$

1.2

$$A: C(x) = 150 + 10x$$

$$B: C(x) = 165 + 7x$$

$$a) A=B \quad 150 + 10x = 165 + 7x$$

$$3x = 15$$

$$x = \cancel{15} 5 \text{ hrs}$$

$$b) A: C(8) = 150 + 10(8) = 230$$

$$B: C(8) = 165 + 7(8) = 221 \quad \leftarrow \text{cheaper!}$$

17) $x = \# \text{ of manuals}$

$$\text{in house: } C(x) = 10,000 + .5x$$

$$\text{outside: } C(x) = .75x$$

$$a) \text{In} = \text{Out} \quad 10,000 + .5x = .75x$$

$$10,000 = .25x$$

$$40,000 = x \quad \text{copies}$$

$$b) \text{in: } C(50,000) = 35,000 \quad \leftarrow \text{cheaper!}$$

$$\text{out: } C(50,000) = 37,500$$

$$28) a) S = A$$

$$21 + .00085x = 43 + .00045x$$

$$.0004x = 22$$

$$x = 55,000 \text{ miles}$$

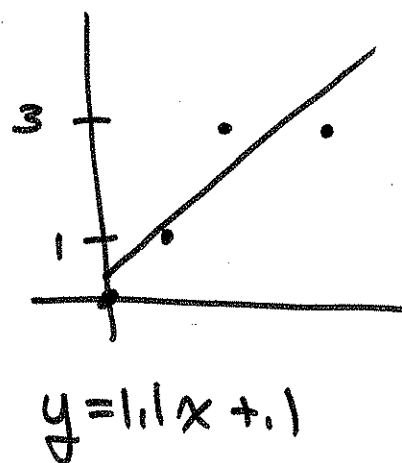
$$b) S(60,000) = 72$$

$$A(60,000) = 70$$

so A has
least CO_2

1.5/ 3, 4, 10, 19, 28, 31, 25

3)	x	y	x^2	xy	y^2
	0	0	0	0	0
	1	1	1	1	1
	2	3	4	6	9
	3	3	9	9	9
Σ	6	7	14	16	19

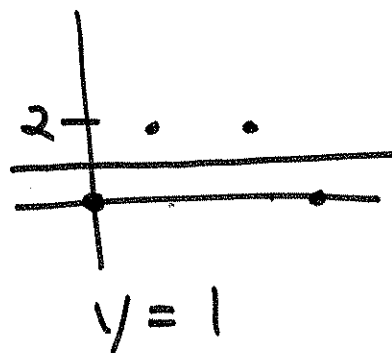


$$M = \frac{4(16) - 6(7)}{4(14) - 6^2} = 1.1 \quad b = \frac{7 - 1.1(6)}{4} = .1$$

$$r = \frac{4(16) - 6(7)}{\sqrt{(4 \cdot 14 - (6)^2)(4 \cdot 19 - (7)^2)}} = .95 \quad \text{good fit}$$

4)

x	y	x^2	xy	y^2	
0	0	0	0	0	
1	2	1	2	4	
2	2	4	4	4	
3	0	9	0	0	
Σ	6	4	14	6	8



$$M = \frac{4(6) - 6(4)}{\text{who cares}} = 0 \quad b = \frac{4 - 0}{4} = 1 \quad r = \frac{4(6) - 6(4)}{\text{who cares}} = 0 \quad \text{no fit}$$

10)

x	y	x^2	xy
3	1.5	9	4.5
5	1.8	25	9
6	2	36	12
13	3.2	169	41.6
18	4.75	324	85.5

$$\Sigma \quad 45 \quad 13.25 \quad 563 \quad 152.6$$

$$b = \frac{5(152.6) - 45(13.25)}{4(56.3) - (45)^2}$$

$$= .211$$

$$b = \frac{13.25 - .211(45)}{5}$$

$$= .751$$

so $y = .211x + .751$

$y = 3 \quad 3 = .211x + .751 \Rightarrow x = 10.659$

so 10,659 thousand

See back of book for 19, 31, 25

28) $x = \# \text{ of yrs past 2000}$

x	y	x^2	xy	y^2
0	172.2	0	0	29652.84
1	177.1	1	177.1	31364.41
2	179.9	4	359.8	32364.01
3	184	9	552	33856
4	188.9	16	755.6	35683.21

$$\Sigma \quad 10 \quad 902.1 \quad 30 \quad 1844.5 \quad 162920.47$$

$$M = \frac{5(1844.5) - 10(902.1)}{5(30) - 10^2} = 4.03$$

$$b = \frac{902.1 - 4.03(10)}{5} = 172.36$$

a) $y = 4.03x + 172.36$

b) in 2006 $x = 6$ $y = 4.03(6) + 172.36 = 196.54$

c) $r = \frac{5(1844.5) - 10(902.1)}{\sqrt{(5 \cdot 30 - 10^2)(5 \cdot 162920.47 - 902.1^2)}}$
 $= .9964 \quad (\text{good fit})$

3.1/ 2, 4, 5, 10

	V	W	R
2) # Light (x)	5	100	20
# Heavy (y)	5	200	30
available	400	10000	

maximize $20x + 30y = \text{Revenue}$

Subject to $5x + 5y \leq 400$

$100x + 200y \leq 10000$

$x \geq 2y$

$x \geq 0$

$y \geq 0$

(# light greater than 2x heavy)

	C	E	Chol
4) Food I (x)	40	10	30
Food II (y)	20	20	20
available	500	200	

minimize

~~maximize~~ $30x + 20y = \text{Chol}$

Subject to $40x + 20y \geq 500$

$10x + 20y \geq 200$

$x + y \leq 30$

$x \geq 0, y \geq 0$

(total at most 30)

5)

	L	W	F	Profit
# of T (x)	1	10	2	4
# of D (y)	3	20	1	3
available	45	350	55	

maximize profit = $4x + 3y$

Subject to $x + 3y \leq 45$
 $10x + 20y \leq 350$

$$2x + y \leq 55$$

$$x \geq 0, y \geq 0$$

10) # of MC acres = $x \rightarrow 2x$ tons

of OC acres = $y \rightarrow 4y$ tons

maximize Profit = $200(2x) + 100(4y)$

subject to

$$x + y \leq 10$$

(total acres)

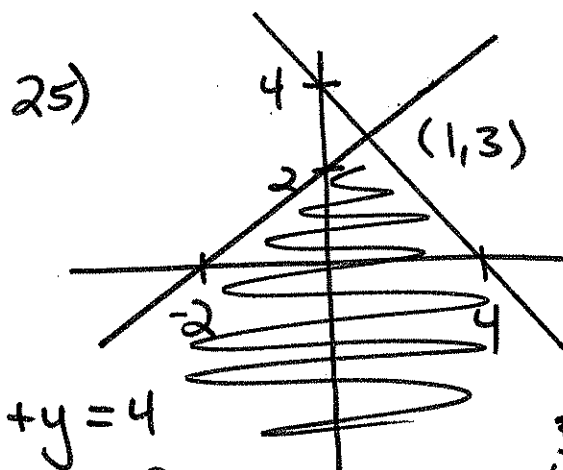
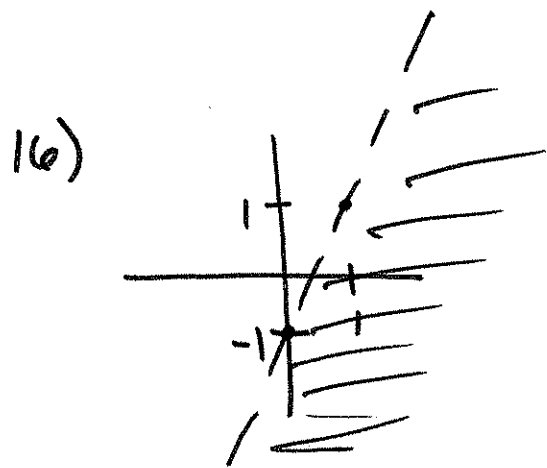
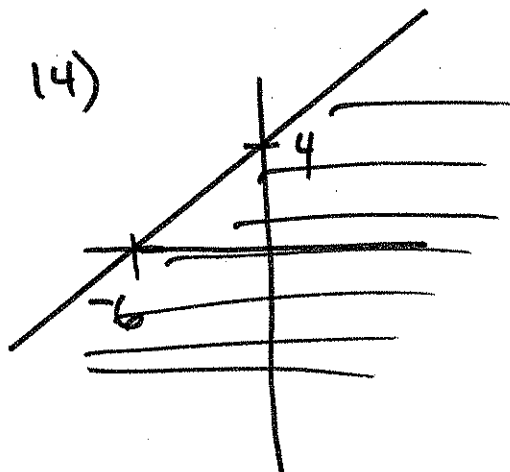
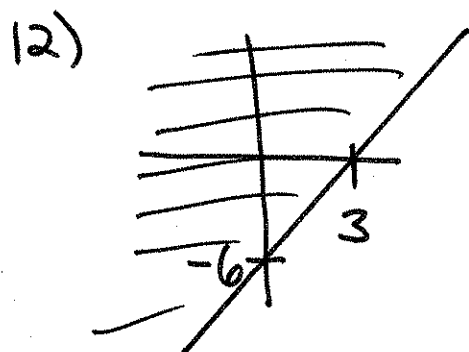
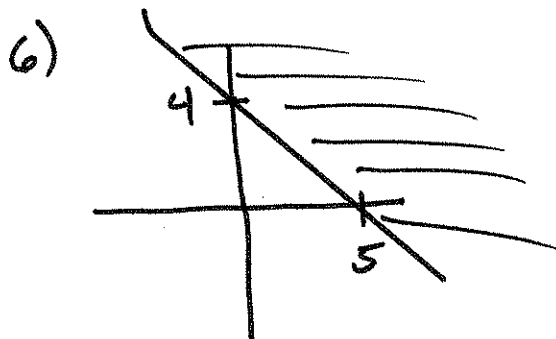
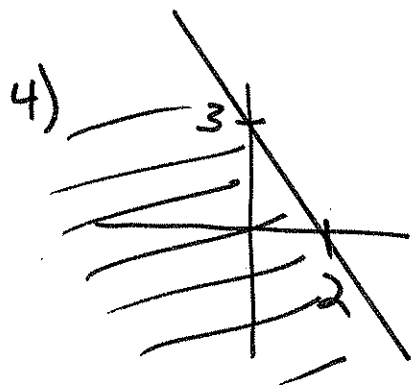
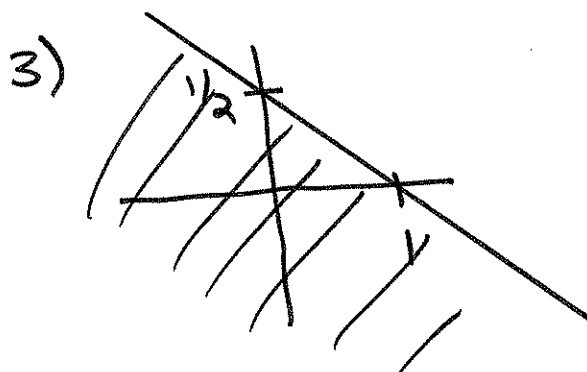
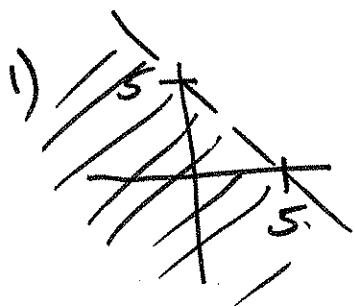
$$2x \geq 3(4y)$$

(tons of MC is
three times more
than tons of OC)

$$x \geq 0$$

$$y \geq 0$$

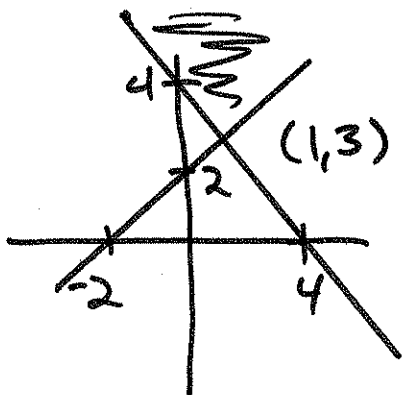
3.2/1, 3, 4, 6, 12, 14, 16, 25, 26, 27, 41, 42, 43



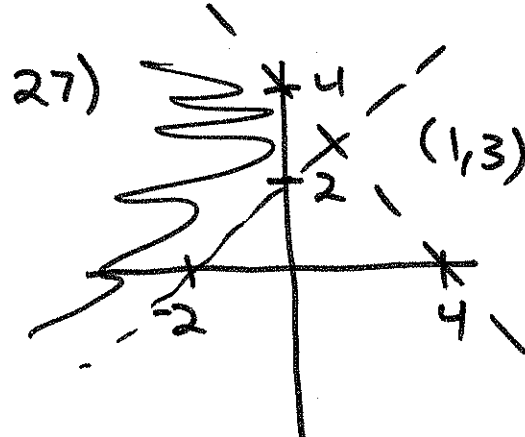
$$\begin{array}{r} x+y=4 \\ -x+y=2 \\ \hline 2y=6 \end{array}$$

$$\begin{array}{r} x+y=4 \\ x+3=4 \\ \hline \text{so } y=3 \end{array} \quad \begin{array}{r} x+y=4 \\ -x+y=2 \\ \hline \text{so } x=1 \end{array}$$

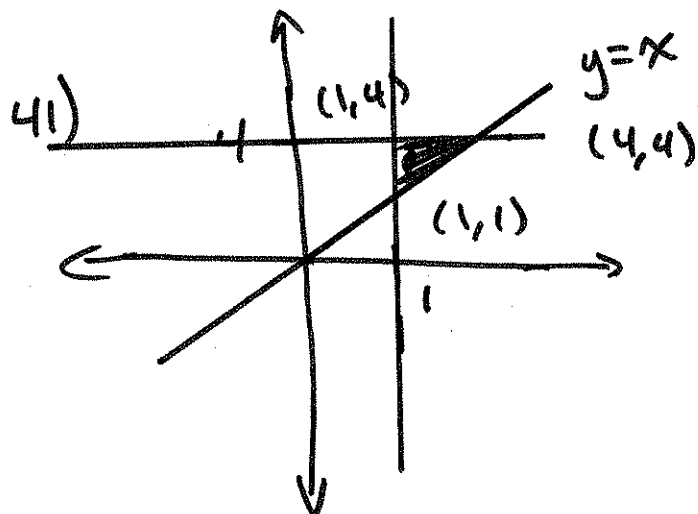
26)



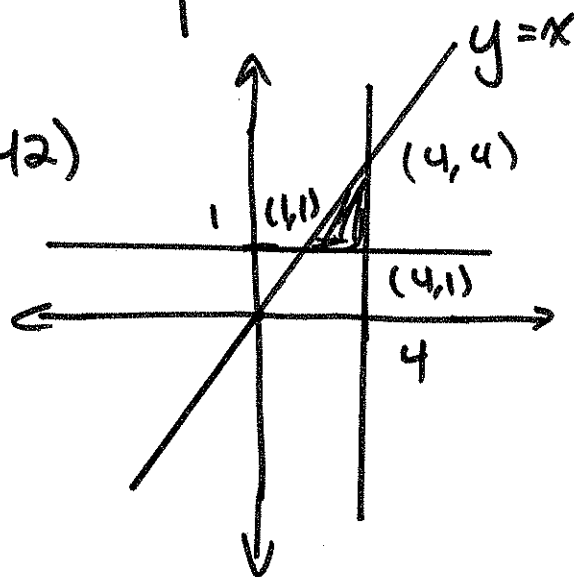
27)



3. 2



42)



43) System

$$R. 2y - x \geq 0$$

$$B. 12y + 5x \leq 44$$

$$G. 8y + 43x \geq -16$$

a) Rand B

$$2y - x = 0 \Rightarrow 2y = x$$

$$12y + 5x = 44$$

$$12y + 10y = 44$$

$$22y = 44$$

$$y = 2$$

$$2 \cdot 2 = x$$

$$x = 4$$

$$(4, 2)$$

b) R and G

3.2

$$2y - x = 0 \Rightarrow 2y = x$$

$$8y + 43x = -16 \Rightarrow 8y + 43(2y) = -16$$

$$8y + 86y = -16$$

$$94y = -16$$

$$y = -16/94 = -8/47$$

$$x = 2y \Rightarrow x = 2(-8/47) = -16/47$$

$$\text{so } (-16/47, -8/47)$$

c) B and G

$$12y + 5x = 44 \longrightarrow 12y = 44 - 5x$$

$$8y + 43x = -16 \quad y = 44/12 - 5/12x$$
$$= 11/3 - 5/12x$$

$$\text{so } 8(11/3 - 5/12x) + 43x = -16$$

$$88/3 - 10/3x + 43x = -16$$

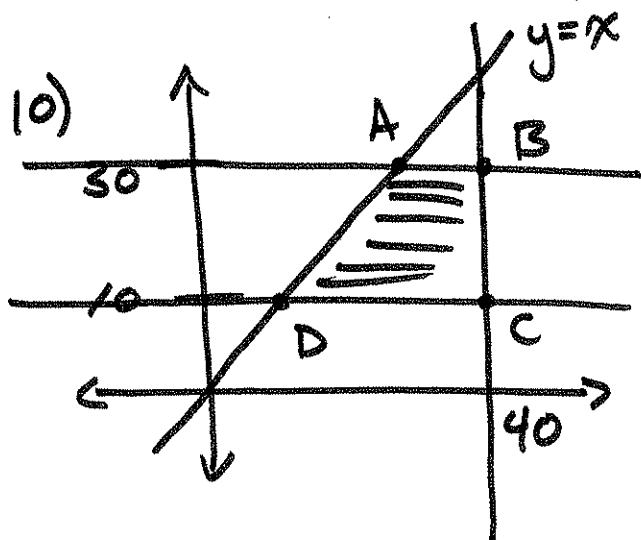
$$88 - 10x + 129x = -48$$

$$119x = -136$$

$$x = -136/119 = -8/7$$

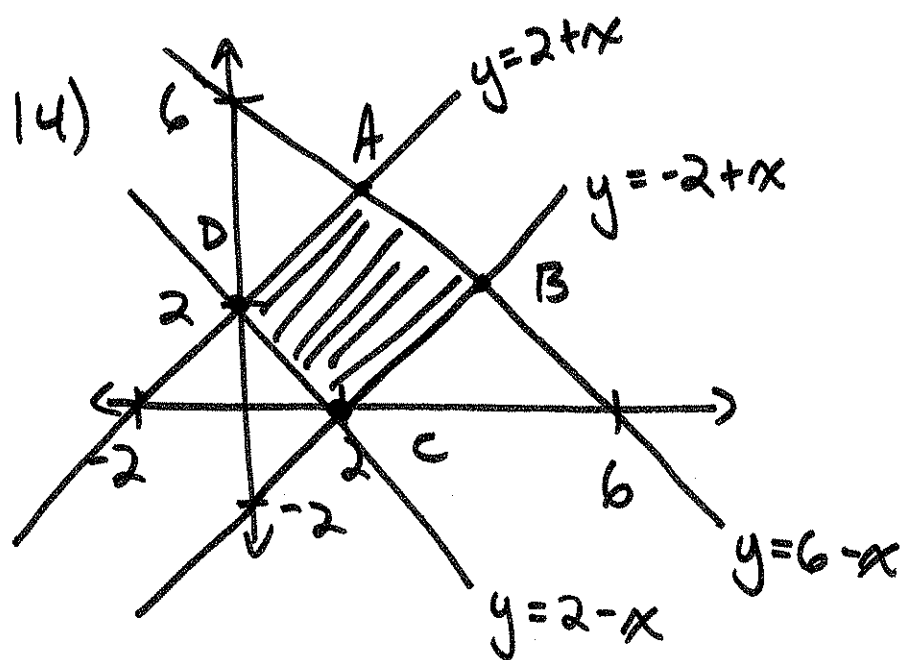
$$\text{so } y = 11/3 - 5/12(-8/7) = 29/7 \quad (-8/7, 29/7)$$

3.3/ 10, 14, 16, 26, 30, 34



		z
A	(30,30)	-30
B	(40,30)	-10
C	(40,10)	50
D	(10,10)	-10

C gives max

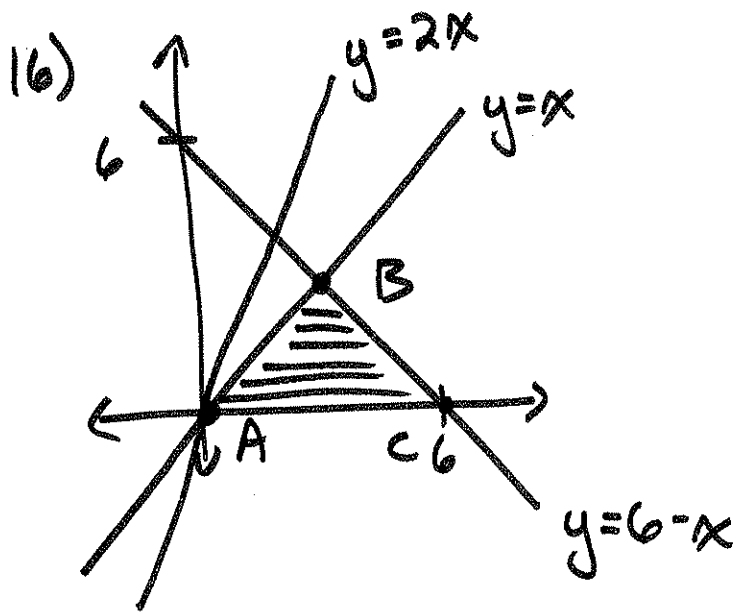


		z
A	(2,4)	28
B	(4,2)	26
C	(2,0)	8
D	(0,2)	10

C gives min

for A $2+x=6-x$
 $2x=4$
 $x=2$
 $y=2+2=4$

for B $-2+x=6-x$
 $2x=8$
 $x=4$
 $y=-2+4=2$



	z
A (0,0)	0
B (3,3)	27
C (6,0)	30

C is max

for B $x=6-x$

$$2x=6$$

$$x=3$$

$$y=x \rightarrow y=3$$

26) See Sec 3.1/4 for equations

$$2x+y \geq 25$$

$$x+2y \geq 20$$

$$x+y \leq 30$$

$$\text{Chol} = 30x + 20y$$

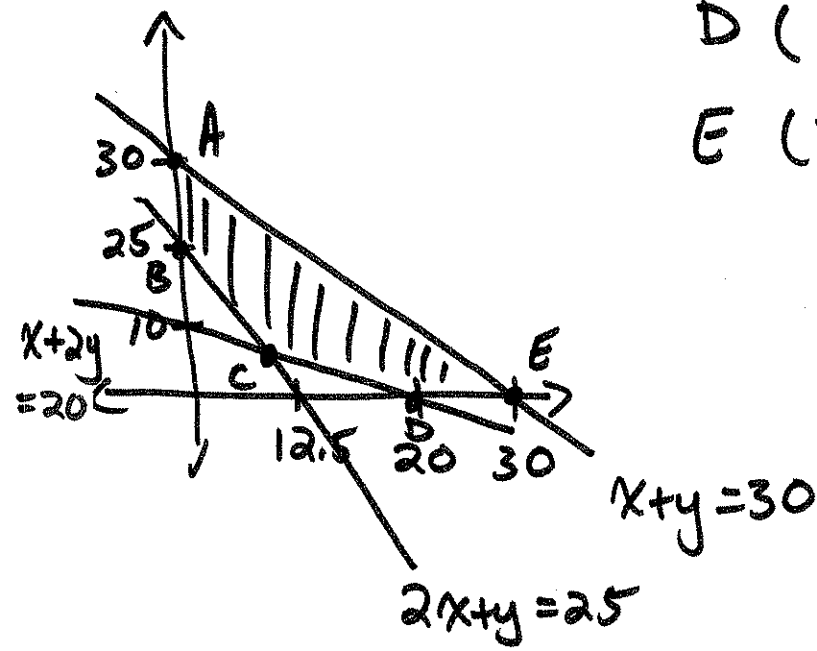
$$A (0,25) \quad 500$$

$$B (0,30) \quad 600$$

$$C (10,5) \quad 400 \quad \text{minimum}$$

$$D (20,0) \quad 600$$

$$E (30,0) \quad 900$$



to get C

$$y=25-2x \quad x=20-2y$$

$$y=25-2(20-2y)$$

$$=25-40+4y$$

$$-3y = -15 \rightarrow y=5$$

30)	P_1	P_2	P_3	Cost
$A(x)$	30	30	10	3000
$B(y)$	10	20	30	4000
Avail.	240	390	410	

Minimize $C = 3000x + 4000y$

Subject to $30x + 10y \geq 240$

$30x + 20y \geq 390$

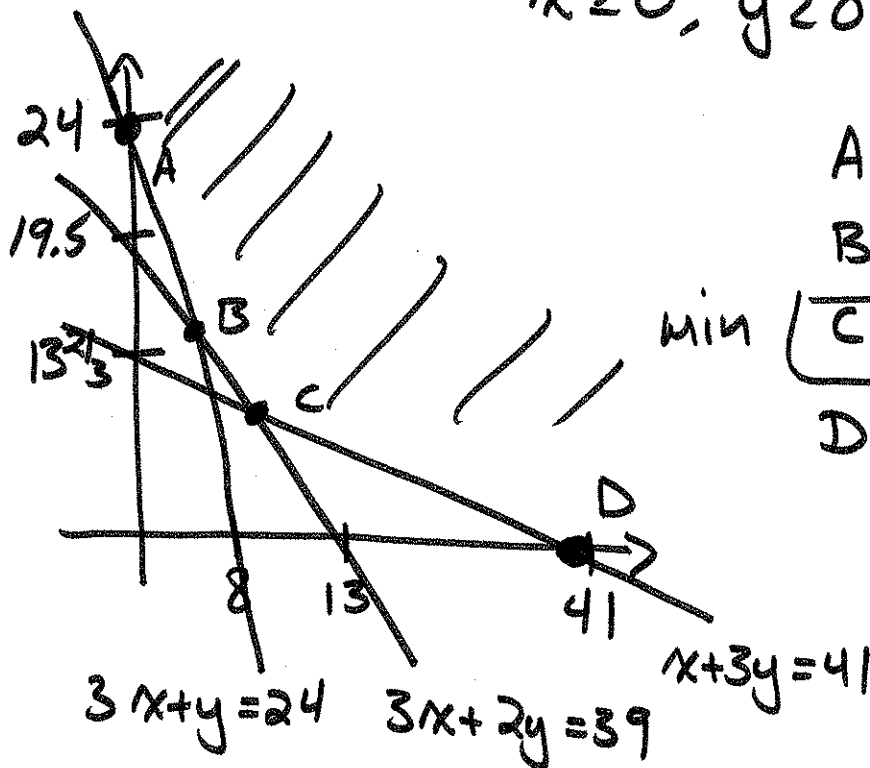
$10x + 30y \geq 410$

$x \geq 0, y \geq 0$

$3x + y \geq 24$

$3x + 2y \geq 39$

$x + 3y \geq 41$



Cost (in 1000's)

$A(0, 24)$

96

$B(3, 15)$

69

$C(5, 12)$

63

$D(41, 0)$

123

Min

for B $y = 24 - 3x$
 $y = -\frac{3}{2}x + 19.5$

for C $y = -\frac{x}{3} + \frac{41}{3}$
 $y = -\frac{1}{3}x + 13.7$

34)

	$P_1(x)$	$P_2(y)$	Avail
C	2	1	30000 (in 1000's)
P	2	3	

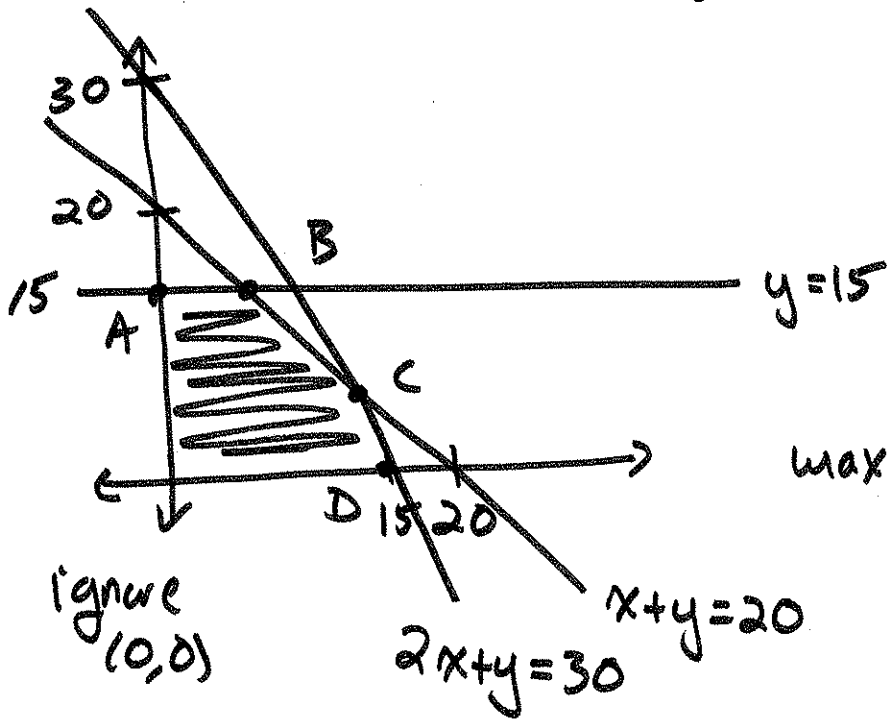
maximize $P = 2x + 3y$

Subject to $2x + y \leq 30$ (in 1000's)

$x + y \leq 20$ (max of both)

$y \leq 15$ (15 of P_2)

$x \geq 0, y \geq 0$



		P
A	(0,15)	45
B	(5,15)	55
C	(10,10)	50
D	(15,0)	30

max

for C

$$\begin{array}{r} 2x + y = 30 \\ - \quad x + y = 20 \\ \hline x = 10 \\ \text{so } y = 10 \end{array}$$

4.1/ 6, 11-14, 31-34, 44, 23-30

6) all T

11) a) 2 b) 3 disjoint

12) a) 4 b) 1 disjoint

13) a) 1, 2, 3 b) 1 not disjoint

14) a) 1, 2, 4 b) 1, 2, 4 so same set!

31) a) $A^c = \{x \mid x \text{ doesn't own an } A\}$ all x 's are state residents
b) $A \cup H = \{x \mid x \text{ owns } A \text{ or } H\}$

c) $A \cup H^c = \{x \mid x \text{ owns } A \text{ or doesn't own } H\}$

32) a) $H^c = \{x \mid x \text{ doesn't own an } H\}$

b) $A \cap H = \{x \mid x \text{ owns both } A \text{ and } H\}$

c) $A^c \cap H = \{x \mid x \text{ doesn't own } A \text{ and owns } H\}$

33) a) $A \cap H^c = \{x \mid x \text{ owns } A \text{ and doesn't own } H\}$ ^{4/1}

b) $A^c \cap H^c = \{x \mid x \text{ doesn't own either } A \text{ or } H\}$

c) $A^c \cup H^c = \{x \mid x \text{ doesn't own } A \text{ or } x \text{ doesn't own } B\}$

34) a) $(A \cap H)^c = 33c$

b) $(A \cup H)^c = 33b$

c) $(A^c \cap H^c)^c = \{x \mid x \text{ owns } A \text{ or } H\}$

44) a) $F \cap (H \cup S)$ or $(F \cap H) \cup (F \cap S)$

b) $N \cap F \cap H$

23) a) $\{4, 5, 6\}$ b) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

24) a) $\{7, 8, 9, 10\}$ b) $\{7, 8\}$

25) a) $\{1, 2, 3\}$ b) $\{9, 10\}$

26) a) $\{1, 2, 3, 7, 8, 9, 10\}$ b) $\{4, 5, 6\}$

* 28) a) $\{4, 5, 6\}$ b) $\{1, 2, 3, 5, 6\}$

~~28~~ * 27) a) $\{5, 6\}$ b) $\{1, 2, 3, 4, 7, 8, 9, 10\}$

29) a) $\emptyset$ b) $\emptyset$

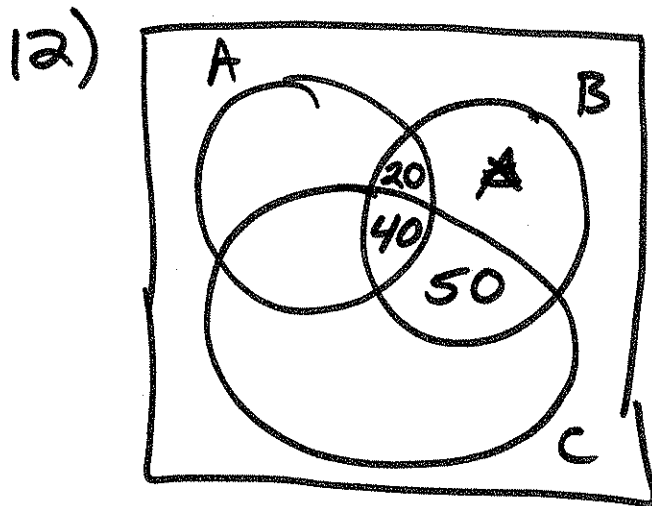
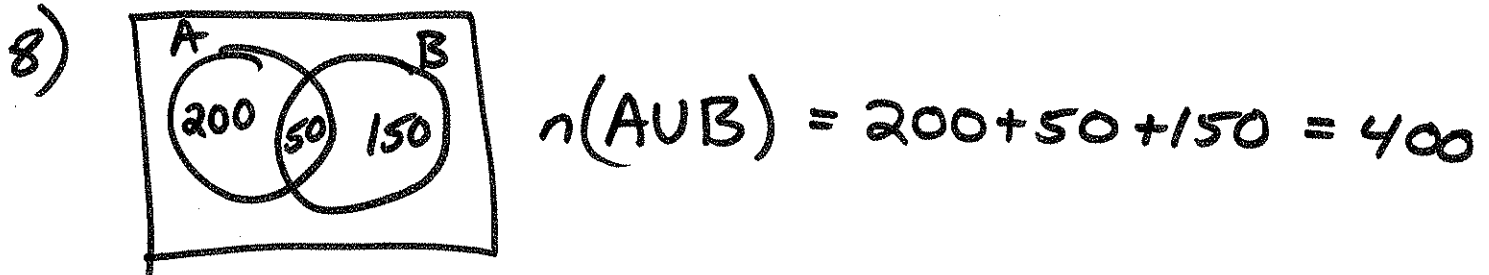
30) a) $\{9, 10\}$ b) $\emptyset$

4.2/2, 8, 12, 18, 24

$$2) \quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$250 = 200 + 100 - n(A \cap B)$$

$$n(A \cap B) = 50$$



* is $A^c \cap B \cap C^c$

Put $n(A \cap B \cap C) = 40$
in first

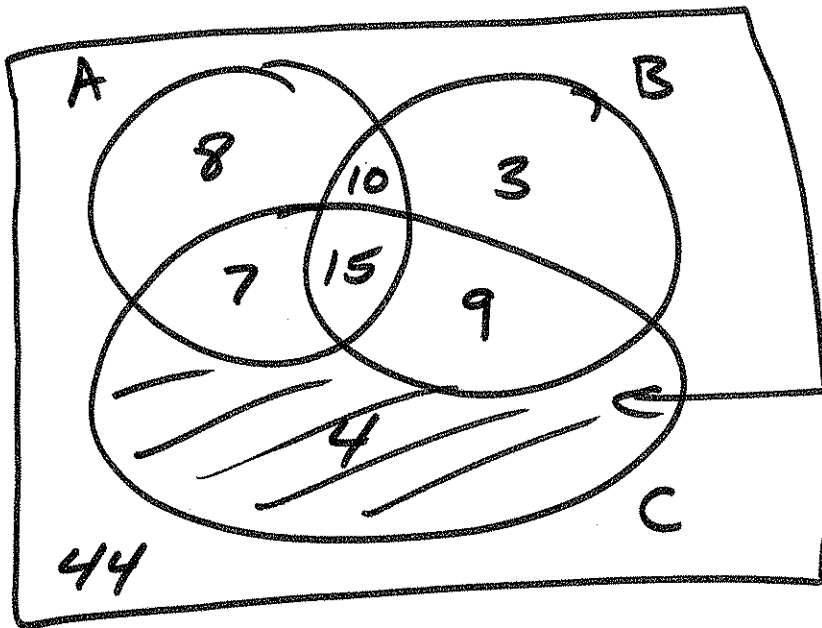
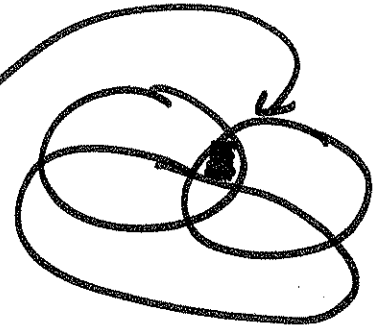
~~use~~

$$n(B) = n(*) + 20 + 40 + 50$$

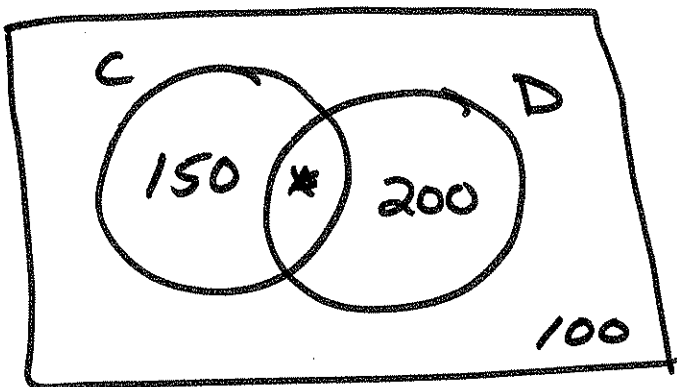
$$200 = n(*) + 110$$

$$90 = n(*)$$

18)

is $A^c \cap B^c \cap C$ start w/ $n(A \cap B \cap C^c) = 10$ then use $n(A \cap B) = 25$ to get center
.... keep goingso $A^c \cap B^c \cap C = 4$ 

24)



$$n(D \cap C^c) = 200$$

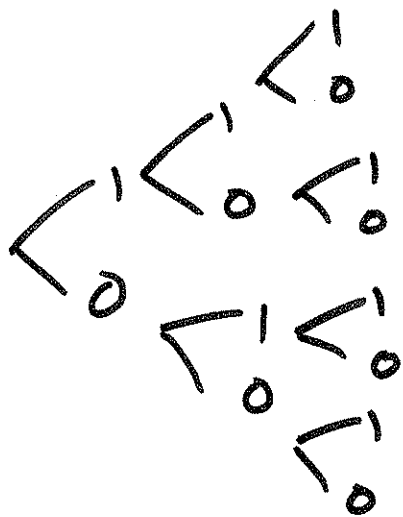
$$n(C \cap D^c) = 150$$

$$n(*) = 500 - (150 + 200 + 100) \\ = 50$$

so $50 + 200 = 250$ own a clog

4.3/ 5, 7, 9, 12, 16, 20

5)



$$S = \{ (1, 1, 1), (1, 1, 0), (1, 0, 1), (1, 0, 0), (0, 1, 1), (0, 1, 0), (0, 0, 1), (0, 0, 0) \}$$

$$E = \{\text{me observed at least twice}\}$$

$$= \{(1, 1, 1), (1, 1, 0), (1, 0, 1), (0, 1, 1)\}$$

7) $S = \{W, B, R\}$

$$E = \{ \text{ball not white} \} = \{ B, R \}$$

9) $S = \{ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE\}$
[is $C(S, 3)$]

$$E = \{ \text{Bubba selected} \}$$

$$= \{ABC, ABD, ABE, BCD, BCE, BDE\}$$

$$12) S = \{C, O, N, E, T, I, U\}$$

$$E = \{O, E, I, U\} = \{\text{vowel is selected}\}$$

16) a) $E \cap F$ = life of light bulb is
between 100, 200 inclusive

b) F^c = life of l.b. is greater than
200 hrs

c) $E^c \cap F$ = life of l.b. is strictly less
than 100 hrs

d) $(E \cup F)^c = \emptyset$ since $E \cup F$ is all
light bulbs

20) not F is F^c

not E is E^c

not F or not E is $F^c \cup E^c$

4.4/5-10, 16, 17, 22, 26, 33, 35

5-10 $n(S) = 52$

5) $E = \{AH, AD, AC, AS\}$ $P(E) = 4/52 = 1/13$

6) $E = \{13 \text{ spades}\}$ $P(E) = 13/52 = 1/4$

7) $E = \{26 \text{ red cards}\}$ $P(E) = 26/52 = 1/2$

8) $E = \{\text{all 3's, 4's, 5's}\}$ $P(E) = 12/52 = 3/13$

9) $E = \{5C, 6C, 7C, 5S, 6S, 7S\}$ $P(E) = 6/52 = 3/26$

10) $E = \{8H, 8D\}$ $P(E) = 2/52 = 1/26$

16) $\frac{5050}{10000} = .505$

17) a) $60/500 = .12$

b) $400/500 = .8$

c) $40/500 = .08$

500 total
60 going < 55
400 between
55 and 65
40 over 65

22) Total sold = $20 + 60 + 100 + 70 = 250$

A) $20/250 = .08$

B) $60/250 = .24$

C) $100/250 = .4$

D) $70/250 = .28$

26) $S = \{ H1, H2, H3, H4, H5, H6, \\ T1, T2, T3, T4, T5, T6 \}$

$E = \{ H2, H4, H6 \}$

$P(E) = \frac{n(E)}{n(S)} = \frac{3}{12} = 1/4$

33) $n(S) = 36$ (see p 202)

a) ~~a 2~~? $E = \{ (1,1) \}$ $P(E) = 1/36$

b) a 4? $E = \{ (3,1), (1,3), (2,2) \}$ $P(E) = 3/36 = 1/12$

c) an 8? $E = \{ (4,4), (5,3), (3,5), (6,2), (2,6) \}$

$P(E) = 5/36$

35) $n(\overset{\text{even}}{\text{vowels}}) = 3$ $n(\text{total}) = 9$ $P(E_{\text{even}}) = 3/9 = 1/3$

5.1/ 2, 4, 6, 16, 18, 20, 28, 33, 35, 40

$$2) P(5,2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3!}{3!} = 5 \cdot 4 = 20$$

$$4) P(8,3) = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 8 \cdot 7 \cdot 6 = 336$$

$$6) P(7,1) = \frac{7!}{(7-1)!} = \frac{7!}{6!} = \frac{7 \cdot 6!}{6!} = 7$$

$$16) \frac{5}{D} \cdot \frac{2}{C} \cdot \frac{2}{C} = 208$$

$$18) \underline{26} \underline{25} \underline{24} \underline{23} \underline{22} \quad \text{or } P(26,5) = 7893600$$

$$20) \frac{4}{H} \cdot \frac{3}{G} \cdot \frac{4}{D} \cdot \frac{5}{C} = 240$$

$$28) 4! = 24$$

$$33) \frac{3!}{\text{out}} \cdot \frac{4!}{\text{in}} = 144$$

$$35) P(12,4)$$

5.1

$$\text{or } \frac{12}{P} \cdot \frac{11}{VP} \cdot \frac{10}{T} \cdot \frac{9}{S} = 11880$$

40)

$$\frac{\frac{8 \cdot 7}{1 \cdot 2}}{\text{Race 1}} \cdot \frac{\frac{10 \cdot 9}{1 \cdot 2}}{\text{Race 2}} = 5040$$

5, 2/3, 5, 7, 19, 23, 25, 29, 33, 38, 40, 42

$$\begin{aligned} 3) C(8,5) &= \frac{8!}{5!(8-5)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{5! 3!} = \frac{8 \cdot 7 \cdot 6}{3!} \\ &= \frac{8 \cdot 7 \cdot 6}{6} = 56 \end{aligned}$$

$$5) C(12,1) = \frac{12!}{1!11!} = \frac{12 \cdot 11!}{1 \cdot 11!} = 12$$

$$7) C(7,4) = \frac{7!}{4!3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3!} = \frac{7 \cdot 6 \cdot 5}{3!} = \frac{7 \cdot 6 \cdot 5}{6} = 35$$

$$19) C(20,5) = 15504$$

$$23) C(40,5) = 658008$$

$$25) C(11,3) \cdot C(7,2) = 3465$$

↙
math

↘
CSci

$$29) H A B \text{ 3 to P.H (9 left)}$$

$$12 \cdot 11 \cdot 10 \cdot C(9,3) = 110880$$

$$33) C(9,5) = 126$$

$$38) C(12,7) = 792$$

$$40) \text{ NYC Dal Den}$$

$$C(10,3) \cdot C(8,3) \cdot C(6,4) = 176400$$

$$42) C(4,3) \cdot C(13,1) \cdot C(4,2) \cdot C(12,1)$$

↓
~~same~~
 rank
 for 3's

↓
 rank for 2's

$$= 3744$$