

Solutions to Practice Problems for Final Exam

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THERE MAY BE ERRORS OR TYPOS!!
SEE YOUR INSTRUCTOR WITH ANY
QUESTIONS!!!

1) $F = P(1+i)^n = 10,000 \left(1 + \frac{.04}{2}\right)^{2 \cdot 4} = 11,716.59$ (b)

2) $P(12,2) \cdot C(10,4) = 27720$ (d)

↳ order matters for
chair and secretary

↳ order doesn't matter
for 4 non-board members

3) use Gaussian elimination to reduce

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & -1 & 0 & 0 \\ 0 & 2 & 2 & 4 & 2 & -4 \\ 0 & 3 & 3 & 5 & 3 & -5 \end{array} \right] \xrightarrow{\frac{1}{2}R_2 \rightarrow R_2} \left[\begin{array}{ccccc|c} 1 & 2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & -2 \\ 0 & 3 & 3 & 5 & 3 & -5 \end{array} \right]$$

$$\xrightarrow{-3R_2 + R_3 \rightarrow R_3} \left[\begin{array}{ccccc|c} 1 & 2 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 2 & 1 & -2 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{ccccc|c} \textcircled{1} & 2 & 1 & -1 & 0 & 0 \\ 0 & \textcircled{1} & 1 & 2 & 1 & -2 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{array} \right]$$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

all rows have leading ones, more variables
than equations $\Rightarrow$ infinitely many solutions (b)

$$x_5 = t$$

$$x_4 = -1$$

$$x_3 = s$$

$$x_2 + x_3 + 2x_4 + x_5 = -2$$

$$x_2 + s - 2 + t = -2$$

$$x_2 = -s - t$$

$$x_1 + 2x_2 + x_3 - x_4 = 0$$

$$x_1 + 2(-s - t) + s + 1 = 0$$

$$x_1 - 2s - 2t + s + 1 = 0$$

$$x_1 = -1 + 2t + s$$

$$4) r_{\text{eff}} = (1+i)^M - 1 = \left(1 + \frac{0.05}{12}\right)^{12} - 1 = 1.05507\% \quad \textcircled{d} \quad \text{L2}$$

$$5) \begin{array}{l} x = \# \text{ Adults} \\ y = \# \text{ Children} \end{array} \quad \begin{array}{l} x + y = 350 \\ 8.50x + 5.50y = 2711 \end{array} \rightarrow \begin{array}{l} x + y = 350 \\ 850x + 550y = 271100 \end{array}$$

$$A = \begin{bmatrix} 1 & 1 \\ 850 & 550 \end{bmatrix} \quad \text{Find inverse} \quad \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 850 & 550 & 0 & 1 \end{array} \right] \rightarrow$$

$$-850R_1 + R_2 \rightarrow R_2$$

$$\left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -300 & -850 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 85/30 & -1/300 \end{array} \right] \rightarrow$$

$$-1/300 R_2 \rightarrow R_2 \quad -R_2 + R_1 \rightarrow R_1$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -55/30 & 1/300 \\ 0 & 1 & 85/30 & -1/300 \end{array} \right] \quad \text{so } A^{-1} = \begin{bmatrix} -55/30 & 1/300 \\ 85/30 & -1/300 \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} -55/30 & 1/300 \\ 85/30 & -1/300 \end{bmatrix} \begin{bmatrix} 350 \\ 271100 \end{bmatrix} = \begin{bmatrix} 262 \\ 88 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

so 262 adults and 88 children

$$8) A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 4 & 8 & 12 \\ 3 & 9 & 18 & 18 \end{bmatrix} \xrightarrow{-1/3 R_1 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 4 & 8 & 12 \\ 0 & 3 & 9 & 6 \end{bmatrix}$$

$$\begin{array}{l} 1/4 R_2 \rightarrow R_2 \\ 1/3 R_3 \rightarrow R_3 \end{array} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 3 & 2 \end{bmatrix} \xrightarrow{-R_2 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{array}{l} \text{so } x + 2y + 3z = 4 \\ y + 2z = 3 \\ z = -1 \end{array}$$

$$\begin{array}{l} y + 2(-1) = 3 \\ y = 5 \\ x + 2(5) + 3(-1) = 4 \\ x = -3 \end{array}$$

$$(x, y, z) = (-3, 5, -1)$$

b) $AX=B$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 8 \\ 3 & 9 & 18 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

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Find A^{-1} $\begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 4 & 8 & | & 0 & 1 & 0 \\ 3 & 9 & 18 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{-1/3 R_1 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 4 & 8 & | & 0 & 1 & 0 \\ 0 & 3 & 9 & | & -3 & 0 & 1 \end{bmatrix}$

$$\xrightarrow{\begin{matrix} 1/4 R_2 \rightarrow R_2 \\ 1/3 R_3 \rightarrow R_3 \end{matrix}} \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & 0 & 1/4 & 0 \\ 0 & 1 & 3 & | & -1 & 0 & 1/3 \end{bmatrix} \xrightarrow{-R_2 + R_3 \rightarrow R_3} \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 2 & | & 0 & 1/4 & 0 \\ 0 & 0 & 1 & | & -1 & -1/4 & 1/3 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} -2R_3 + R_2 \rightarrow R_2 \\ -3R_3 + R_1 \rightarrow R_1 \end{matrix}} \begin{bmatrix} 1 & 2 & 0 & | & 4 & 3/4 & -1 \\ 0 & 1 & 0 & | & 2 & 3/4 & -2/3 \\ 0 & 0 & 1 & | & -1 & -1/4 & 1/3 \end{bmatrix} \xrightarrow{-2R_2 + R_1 \rightarrow R_1} \begin{bmatrix} 1 & 0 & 0 & | & 0 & -3/4 & 1/3 \\ 0 & 1 & 0 & | & 2 & 3/4 & -2/3 \\ 0 & 0 & 1 & | & -1 & -1/4 & 1/3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & -3/4 & 1/3 \\ 2 & 3/4 & -2/3 \\ -1 & -1/4 & 1/3 \end{bmatrix} \text{ so } X = A^{-1}B = \begin{bmatrix} 0 & -3/4 & 1/3 \\ 2 & 3/4 & -2/3 \\ -1 & -1/4 & 1/3 \end{bmatrix} \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix} = \begin{bmatrix} -3 \\ 5 \\ -1 \end{bmatrix}$$

7) E1 $5y + 6x \geq 85 \rightarrow (0, 17), (14.2, 0)$

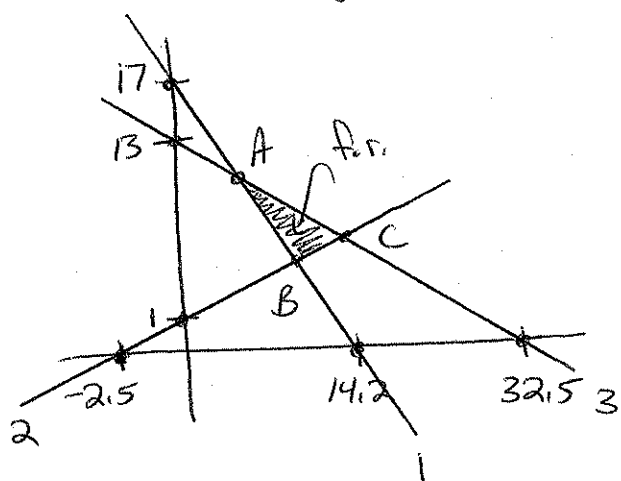
intercepts

a) E2 $5y - 2x \geq 5 \rightarrow (0, 1), (-2.5, 0)$

corner pts circled

E3 $10y + 4x \leq 130 \rightarrow (0, 13), (32.5, 0)$

$x, y \geq 0$



A) $\begin{cases} 5y + 6x = 85 \\ 10y + 4x = 130 \end{cases} \rightarrow \begin{cases} 5y + 6x = 85 \\ -10y - 12x = -170 \end{cases}$

$$-8x = 40$$

$$x = 5$$

$$5y + 30 = 85$$

$$y = 11$$

$$(5, 11)$$

B) $\begin{cases} 5y + 6x = 85 \\ -(5y - 2x = 5) \end{cases}$

$$8x = 80$$

$$x = 10$$

$$5y + 60 = 85$$

$$y = 5$$

$$(10, 5)$$

c) $\begin{cases} 5y - 2x = 5 \\ 10y + 4x = 130 \end{cases} \rightarrow \begin{cases} 5y - 2x = 5 \\ 10y - 4x = 10 \end{cases}$

$$20y = 140$$

$$y = 7$$

$$35 - 2x = 5$$

$$15 = x$$

$$(15, 7)$$

b) pt	$z = 7x + 5y$
(5, 11)	90
(10, 5)	95
(15, 7)	140

max z value

$$8) P(20,3) = \frac{20}{9} \cdot \frac{19}{5} \cdot \frac{18}{6} = 6840$$

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$$9) \frac{C(20,12)}{\text{jurers}} \cdot \frac{C(8,2)}{\text{alternates}} = 3,527,160$$

$$10) \frac{C(26,2) C(26,2)}{C(52,4)} = \frac{105625}{270725} = .39016$$

$$11) p = 1/2 \quad q = 1/2 \quad C(6,3) (1/2)^3 (1/2)^{6-3} = .3125$$

$$12) \mu = 16 \quad \sigma = .5 \quad (\text{using table})$$

$$a) P(X \leq 15) = P(Z \leq \frac{15-16}{.5}) = P(Z \leq -2) = .0228$$

$$b) P(X \geq 17.5) = 1 - P(X \leq 17.5) = 1 - P(Z \leq \frac{17.5-16}{.5}) \\ = 1 - P(Z \leq 3) = 1 - .9987 = .0013$$

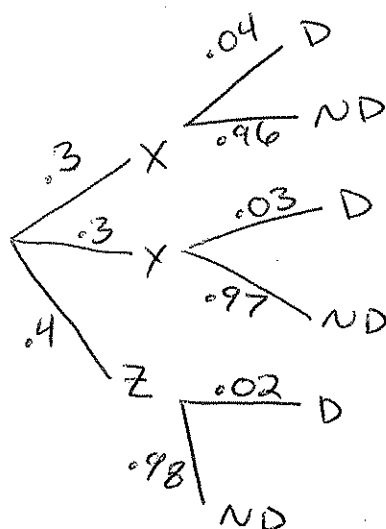
$$c) P(15 \leq X \leq 17.5) = P(X \leq 17.5) - P(X \leq 15) \\ = P(Z \leq 3) - P(Z \leq -2) = .9987 - .0228 = .9759$$

$$13) a) \frac{C(6,3) \cdot C(10,3)}{C(16,6)} = \frac{20 \cdot 120}{8008} = .2997$$

$$b) P(\text{at least 2 R}) = 1 - P(0R) - P(1R) \\ = 1 - \frac{C(6,6)}{C(16,6)} - \frac{C(10,1) C(6,5)}{C(16,6)} = 1 - \frac{1}{8008} - \frac{60}{8008} = .9924$$

14)

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 $P(\text{from } Z \mid \text{def})$

$$= \frac{P(Z \cap D)}{P(D)} = \frac{P(Z \cap D)}{P(Z \cap D) + P(Y \cap D) + P(X \cap D)}$$

$$= \frac{.4(.02)}{.4(.02) + .3(.03) + .3(.04)}$$

$$= .2759$$

15) Bernoulli Trials $p = 9/10$ $q = 1/10$

$$C(20, 18) \left(\frac{9}{10}\right)^{18} \left(\frac{1}{10}\right)^2 + C(20, 19) \left(\frac{9}{10}\right)^{19} \left(\frac{1}{10}\right)^1 + C(20, 20) \left(\frac{9}{10}\right)^{20} \left(\frac{1}{10}\right)^0 = .6769$$

16) a) $(900, 460)$ $m = \frac{700 - 460}{500 - 900} = \frac{240}{-400} = -3/5$
 $(500, 700)$

$$p - 460 = -3/5(x - 900)$$

$$= -3/5x + 540$$

$$p = -3/5x + 1000 \quad \text{demand}$$

b) supply = demand $-3/5x + 1000 = .1x + 300$
 $700 = .7x$
 $1000 = x$

$$x = 1000 \text{ so } p = .1(1000) + 300 = 400$$

1000 tons sold at \$400 per ton

17) $x = \# \text{ units of Food 1}$
 $y = \# \text{ units of Food 2}$

	protein	fat	cost
x	2	4	15
y	6	2	10
needed	30	20	

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Ob. f. Cost = $15x + 10y$

constraints

$$2x + 6y \geq 30$$

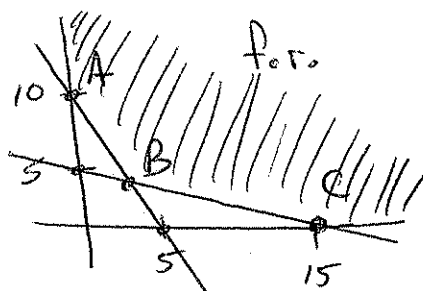
$$4x + 2y \geq 20$$

$$x, y \geq 0$$

$$\rightarrow (0, 5) (15, 0)$$

$$\rightarrow (0, 10) (5, 0)$$

intercepts



Cornerpts $A = (0, 10)$ $C = (15, 0)$

B:
$$\begin{aligned} 2x + 6y &= 30 \\ 4x + 2y &= 20 \\ -2(4x + 2y) &= -40 \\ \hline -10y &= -40 \\ y &= 4 \end{aligned}$$

$2x + 2(4) = 30$
 $2x + 8 = 30$
 $2x = 22$
 $x = 11$

$(3, 4)$

C.P.	Ob. fun.
$(0, 10)$	$15 \cdot 0 + 10 \cdot 10 = 100$
$(3, 4)$	$15 \cdot 3 + 10 \cdot 4 = 85$
$(15, 0)$	$15 \cdot 15 + 10 \cdot 0 = 225$

← minimum cost of 85¢
 for 3 units of Food 1
 4 units of Food 2

18) 36 ways for 2 dice to fall

11	21	31	41	51	61	$x=5$
12	22	32	42	52	62	$x=4$
13	23	33	43	53	63	$x=3$
14	24	34	44	54	64	$x=2$
15	25	35	45	55	65	$x=1$
16	26	36	46	56	66	$x=0$
	$x=5$	$x=4$	$x=3$	$x=2$	$x=1$	$x=0$

along each
 diagonal is the
 same absolute value

c) continued
 $\text{Var} = 2.05$
 $\sigma = \sqrt{V} = \sqrt{2.05}$
 $= 1.43$

a)	x	0	1	2	3	4	5
	$P(X=x)$	$6/36$	$10/36$	$8/36$	$6/36$	$4/36$	$2/36$

b) $E(X) = 0(6/36) + 1(10/36) + 2(8/36) + 3(6/36) + 4(4/36) + 5(2/36)$
 $= 70/36 = 1.94$

c) $\text{Var}(X) = 0^2(6/36) + 1^2(10/36) + 2^2(8/36) + 3^2(6/36) + 4^2(4/36) + 5^2(2/36) - (70/36)^2$