The Unit Circle

We measure angles on the unit circle, in radians (not degrees...radians and degrees are like centimeters and inches. For reasons we won't get into too much, radians just make more sense than degrees. Ask me sometime why a full circle is 360 degrees). An angle of zero radians corresponds to 3:00 on a clock, and positive angles go in the counterclockwise direction. A full circle is 2π radians. In other words, 2π radians is the same measurement as 360 degrees.

1. If a full circle is 2π radians, how much is a half circle?

   1. ______________

2. If a full circle is 2π radians, how much is a quarter circle?

   2. ______________

3. If a full circle is 2π radians, how much is an eighth of a circle?

   3. ______________

4. If we cut a pizza into six slices, what is the angle of each slice? (in radians!)

   4. ______________

5. What is the angle corresponding to 10:00?

   5. ______________

6. On the circle on the last page, mark off the angles $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$, and $\frac{\pi}{2}$.

7. A long time ago, some enterprising souls figured out that if you have a circle of radius one centered at the origin (which we will call the *unit circle*), and you draw a line from the origin at an angle of $\frac{\pi}{6}$ until it hits the unit circle, then the point where the line and circle intersect has the (x, y)-coordinates $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. Add these coordinates to your picture on the next page.

8. These same intelligent people figured out that if the angle is $\frac{\pi}{4}$, then the point has coordinates $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$. Add that to your picture.

9. There's a little bit of symmetry going on. What are the coordinates corresponding to $\frac{\pi}{3}$? Label those as well.

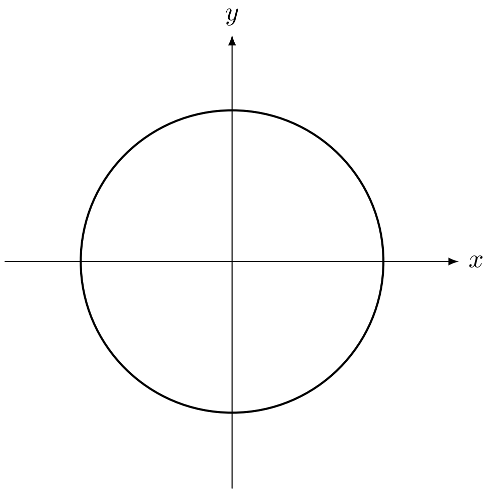
10. Label the coordinates of 0 and $\frac{\pi}{2}$.

11. Now there's even more symmetry going on. For example, if you now find the angles $\frac{2\pi}{3}, \frac{3\pi}{4}$, and $\frac{5\pi}{6}$, you should be able to label their coordinates just using what you've already done. So do that.

12. In fact, finish it up. Add all multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ that you haven't put on there yet, anything less than 2π, and write their coordinates.

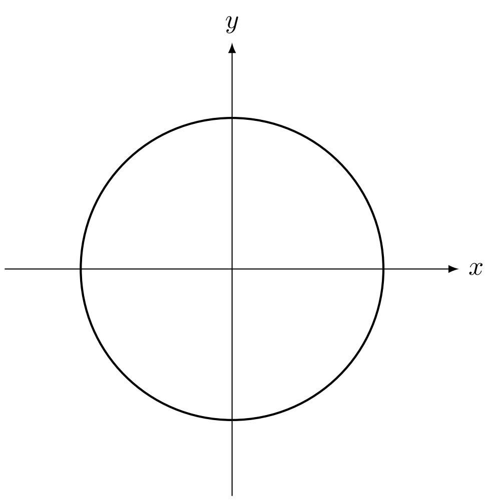
13. Now memorize that picture. Don't waste time memorizing the whole thing, though – instead of memorizing over 30 angles and coordinates, just memorize a few and use symmetry and some logical reasoning to figure out the rest.

14. Of course, there are more angles than the ones you've drawn. Those are just commonly appearing angles. On the circle below, find the approximate locations of $\frac{\pi}{5}$, $\frac{8\pi}{7}$, and $\frac{19\pi}{20}$.



15. We usually use π when we're reporting measurements of radians. Do we have to? Can you find the approximate location of 1 radian on a circle? (In fact, there's a very special relationship between a measure of 1 radian and the radius of a circle...)

16. If positive angles are measured in the counterclockwise direction, negative angles are measured in the clockwise direction. Find $-\frac{\pi}{3}$, $-\frac{7\pi}{6}$, and $-\frac{3\pi}{2}$ on the unit circle below.



17. Two angles are *coterminal* if they lie in the same spot on the unit circle. For example, 0 radians and 2π radians are coterminal, since they're both at 3:00 on the unit circle. Which of the following angles are coterminal?

$$\frac{7\pi}{6} \qquad -\frac{\pi}{6} \qquad \frac{13\pi}{6} \qquad \frac{73\pi}{6}$$

18. Given any angle t on the unit circle, the *reference angle* is the acute angle that is formed by the x-axis and the angle t. For example, the reference angle for $\frac{2\pi}{3}$ is $\frac{\pi}{3}$. What is the reference angle for the following angles?
    (a) $-\frac{5\pi}{6}$
    (b) $\frac{5\pi}{4}$
    (c) $\frac{3\pi}{5}$

My Very Own Special Unit Circle