Solving Trigonometric Equations

The easiest trig equations just involve a good knowledge of the unit circle.

1. Find a value for x such that $\sin(x) = -\frac{\sqrt{2}}{2}$.

2. Find a value for θ such that $\cos(\theta) = \frac{1}{2}$.

3. Find a value for t such that $\tan(t) = -\sqrt{3}$.

In the above, you found *a* solution to those equations. When dealing with trig functions, however, there may be more than one solution. In fact, there's usually an infinite number of solutions. Given an angle θ, we can write all angles that are coterminal with θ as "$\theta + 2\pi k$, for any integer k." For example, if we want to represent the set of angles $\{0, 2\pi, 4\pi, 6\pi, -2\pi, -4\pi, \ldots\}$, we could just write "$0 + 2\pi k, k \in \mathbb{Z}$" (that "$k \in \mathbb{Z}$" stuff is mathematician shorthand for "k is any integer.").

4. Find *all* values of x such that $\sin(x) = -\frac{\sqrt{2}}{2}$.

5. Find *all* values of t such that $\tan(t) = 1$.

6. Find *all* values of θ such that $\csc(\theta) = 1$.

If you have a more complicated trig equation, your main goal is to use algebraic techniques to transform it into something simple, like one of those above.

7. Solve for t: $\sqrt{2}\cos t = -1$.

8. Solve for t: $\dfrac{3 + 2\sin t}{5} = \sin t$.

Sometimes we get tired of writing $+2\pi k$ all the time. A common thing to do is to restrict our attention to solutions that lie in the interval $[0, 2\pi)$.

9. Find all solutions in the interval $[0, 2\pi)$: $1 = \dfrac{1 + 3\cos\theta}{5\cos\theta - 2}$.

10. Find all solutions in the interval $[0, 2\pi)$: $\dfrac{6\sec t + 2}{2\sec t - 1} = 2$.

Sometimes, some more complicated algebraic techniques might be required. Things like factoring, and then using the fact that $AB = 0 \implies A = 0$ or $B = 0$. Things like using the fact that $\sec(x) = \frac{1}{\cos(x)}$, or $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Things like treating $\sin(x)$ as a single "thing" (which it is), and factoring $\sin^2(x) - 2\sin(x) - 3$ exactly the same way you would factor $u^2 - 2u - 3$.

11. Find all solutions in $[0, 2\pi)$: $2\sin^2 t + \sqrt{3}\sin t = 0$ (Try factoring the left hand side.)

12. Find all solutions: $2\sin t\cos t = \sin t$ (Try moving all terms to one side and then factoring.)

13. Find all solutions in $[0, 2\pi)$: $2\cos^2 t + \cos t - 1 = 0$. (Try factoring it like a quadratic.)

14. Find all solutions in $[0, 2\pi)$: $\sin t + \tan t = 0$. (Try rewriting $\tan(x)$, then factoring.)

15. Solve for θ: $2\sin^2\theta - 3\sin\theta + 1 = 0$

16. Solve for x: $\tan x\sec x + \sqrt{2}\tan x = 0$

Sometimes your answers have to be expressed using inverse trig functions, since they won't always work out nicely.

17. Find two solutions for x: $3\cos^2(x) + \cos(x) - 2 = 0$.

What if you had a more complicated expression inside a trig function? Something like $\tan(2x)$? Hint: Let $u = 2x$, solve for u, and then substitute back to solve for x.

18. Find all solutions in $[0, 2\pi)$: $\tan(\frac{x}{2}) = 1$.

19. Find all solutions: $\cos(2x) = -\frac{\sqrt{2}}{2}$.

You can also use trig identities to help out with simplifying equations.

20. Find all solutions to $\sin(2x) = \cos x$.

21. Solve $\sec^2 x - 2\tan x = 4$.

22. Find all solutions in $[0, 2\pi)$ of $2\cot^2(x) + \csc^2(x) - 2 = 0$.