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**Inverse Functions**

1. If I had a function  $f$ , and I told you to “find the opposite of  $f$ ,” there are different things you could come up with. Illustrate the three options with the function  $f(x) = x - 5$ .
  1. The “opposite” could be the negative of  $f$ , which is:
  2. The “opposite” could be the reciprocal of  $f$ , which is:
  3. The “opposite” could be the inverse of  $f$ , which is:
2. Are any of these the same thing?
3. Let  $g$  be the function that takes a number, adds 3 to that number, and then multiplies the result by 42. What does the inverse of  $g$  do?
4. In words, describe what the inverse of a function is.
5. The notation we use for the inverse of a function  $f$  is  $f^{-1}$ . Why is this not very good notation? (although we're stuck with it)
6. Let  $f(x) = x^3 - 5$ . Which of the following is true?
  1.  $f^{-1}(2) = 3$
  2.  $f^{-1}(0) = 2$
  3.  $f^{-1}(-4) = 1$
  4.  $f^{-1}(1) = 4$

7. Let's say  $g(2) = 5$ . Which of the following might be the inverse of  $g$ ?

1.  $g^{-1}(x) = x + 3$
2.  $g^{-1}(x) = x - 3$
3.  $g^{-1}(x) = 2x - 7$
4.  $g^{-1}(x) = x$

8. Let  $h$  be given by the following table:

$x$	-2	-1	0	1	2
$h(x)$	-3	2	0	-2	1

(a) Fill in as much as you can of the table for  $h^{-1}$ . If an entry is undefined, leave it blank.

$x$	-2	-1	0	1	2
$h^{-1}(x)$					

(b) What is  $(h \circ h^{-1})(1)$ ?

(c) What is  $(h^{-1} \circ h)(1)$ ?

9. For any invertible function  $f$ , what is  $(f \circ f^{-1})(x)$ ?

10. For any invertible function  $f$ , what is  $(f^{-1} \circ f)(x)$ ?

11. Is  $\frac{x}{2} + 4$  the inverse of the function  $2x - 8$ ? Check by composing the two functions.

12. Is  $\frac{3}{5-x}$  the inverse of the function  $\frac{3-x}{5}$ ? Check by composing the two functions.

Now we're familiar with inverses and what they are. Let's take a look at inverses from a graphical perspective.

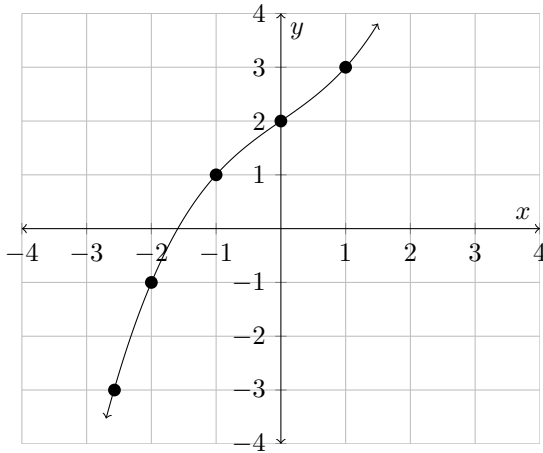
13. Say the point  $(a, b)$  is on the graph of an invertible function  $f$ .

(a) What does that mean in terms of the function  $f$ ? (inputs and outputs)

(b) What does that mean in terms of the inverse function  $f^{-1}$ ?

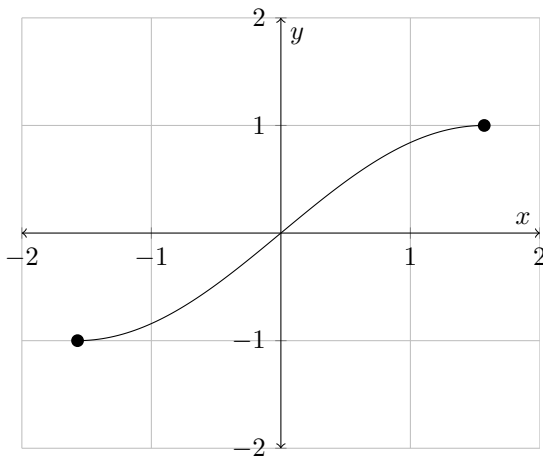
(c) What does that mean about the graph of the inverse function  $f^{-1}$ ?

14. To summarize the previous question, if the point  $(a, b)$  is on the graph of  $f$ , then the point \_\_\_\_\_ is on the graph of  $f^{-1}$ .
15. Here's a graph of a function  $f$ , with some points marked.



- (a) For each marked point, draw the corresponding point on the graph of  $f^{-1}$ .
- (b) Sketch a rough graph of  $f^{-1}$  based on the points you've drawn.
16. What is the relationship between the graph of  $f$  and the graph of  $f^{-1}$ ?

17. Below is a graph of a function. Sketch a graph of its inverse.



Now let's consider inverse functions algebraically.

18. What would be a good procedure for finding the formula for the inverse of a function?

19. Let  $f(x) = \frac{3x-2}{5}$ . Find a formula for the inverse of  $f$ .

20. Let  $g(x) = x^3 - 3$ . Find a formula for the inverse of  $g$ .

21. Let  $h(x) = \frac{3x-2}{x+1}$ . Find a formula for the inverse of  $h$ .