Radicals

n and *m* denote positive integers

☺ **n[th] Root**: $\sqrt[n]{a} = b$ means $b^n = a$

- If *n* is even then: $\sqrt[n]{a}$ is not defined for $a < 0$, and $\sqrt[n]{a}$ is positive for $a > 0$.
- If *n* is odd then**:** $\sqrt[n]{a}$ is always defined, and is uniquely determined by *a*.

☺ **Rational Exponents:**

- $a^{\frac{1}{n}} = \sqrt[n]{a}$
- $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ as long as $\sqrt[n]{a}$ is defined.
- $a^{-\frac{m}{n}} = \dfrac{1}{a^{\frac{m}{n}}}$ as long as $a^{\frac{m}{n}}$ is a nonzero number

All the Exponent Rules that work with integer exponents, also work with rational exponents

☺ **Translation of Some of the Exponent Notation and Rules into Radical Language**:

1. $\sqrt[n]{a^n} = a$ if *n* is an odd, and $\sqrt[n]{a^n} = |a|$ if *n* is an even.

2. $\left(\sqrt[n]{a}\right)^n = a$ as long as $\sqrt[n]{a}$ is defined.

3. $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ as long as $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are defined.

4. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$ as long as $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are defined, and $\sqrt[n]{b}$ is nonzero.