

### Exam 1 (with solutions in red)

**Exercise 1** (6 points) List the elements of the set  $\{-8, -\sqrt{4}, 0, \frac{1}{2}, \sqrt{2}, \pi, 12\}$  that are also elements of the set of Rational Numbers. Justify your answer. An answer with no justification will not receive full credit.

Answer:  $-8, -\sqrt{4} = -2, 0, \frac{1}{2}, 12.656565\dots$

Justification:

Because these are the numbers that can be expressed as fractions (12.656565.... exhibits a repeated pattern after the decimal point)

**Exercise 2** Determine whether each of the following statements is True or False. In each case justify your answer. An answer with no justification will not receive full credit.

**Exercise 2a** (3 points)  $-4^2 = 16$

Answer: False

Justification:

False, because  $-4^2 = -(4^2) = -16$

**Exercise 2b** (3 points)  $\frac{-5}{2} \geq \frac{(-1)(5)}{2-4}$

Answer: False

Justification:

False, because the right side is equal to  $5/2$ , and when plotting both sides of the equation on the real line  $-5/2$  is to the left of  $5/2$ .

**Exercise 3** Write each of the following statements using mathematical symbols.

**Exercise 3a** (3 points) Three times the square of the difference between  $x$  and three is less or equal to twelve.

Answer:  $3(x-3)^2 \leq 12$

**Exercise 3b** (3 points) Five subtracted from the quotient of  $x$  and four is equal to the absolute value of twice the sum of four and minus seven.

Answer:  $\frac{x}{4} - 5 = |2(4 + (-7))|$

**Exercise 4** (6 points) Calculate the following expressions. For full credit show your work.

$$\frac{(1-3)^3 + \sqrt{49}}{6-3 \cdot 12 \div 9}$$

Answer:  $-\frac{1}{2}$

Calculation:

$$\frac{(1-3)^3 + \sqrt{49}}{6-3 \cdot 12 \div 9} = \frac{(-2)^3 + 7}{6-4} = \frac{-8+7}{2} = -\frac{1}{2}$$

**Exercise 5** (6 points) Simplify the following algebraic expressions by using the distributive law to eliminating parentheses and combine like terms. For full credit show your work.

$$\frac{x}{2} + \frac{y}{3} - \frac{1}{2}(x-y)$$

Answer:  $\frac{5y}{6}$

Work:

$$\frac{x}{2} + \frac{y}{3} - \frac{1}{2}(x-y) = \frac{x}{2} + \frac{y}{3} - \frac{1}{2} \cdot x + \frac{1}{2} \cdot y = \frac{x}{2} + \frac{y}{3} - \frac{x}{2} + \frac{y}{2} = \frac{y}{3} + \frac{y}{2} = \frac{2y}{6} + \frac{3y}{6} = \frac{5y}{6}$$

**Exercise 6** (6 points) Evaluate the following algebraic expressions at the given values of variables. For full credit show your work.

$|2y - 5x|$  at the values  $y = 3$  and  $x = 3.2$

Answer: 10

Work:

$$|2 \cdot 3 - 5 \cdot 3.2| = |6 - 16| = |-10| = 10$$

**Exercise 7** (6 points) You are waiting for the big sales at Filenes to buy a coat which is originally priced at \$98. You can afford to pay only \$88 for this coat. This weekend Filenes advertises 10% off every winter cloth item. Can you afford to buy the coat this weekend? For full credit show your work and justify your answer

Set the conversion rectangle to calculate 10% of \$98 :

\$98	-----	100%
\$?	-----	10%

$98 \times 10/100 = \$9.8$ . The price is  $\$98 - \$9.8 = \$88.2$ .

You can afford \$88. Borrow 20 cents from a friend and go for it.

**Exercise 8** The area of a rectangle with length  $L$  and width  $W$  is equal to  $L \cdot W$ .

**Exercise 8a** (6 points) Draw and calculate the area of a rectangle with length equal to 2 inches, and width equal to 4 inches. Specify the units of the area. For full credit show your work.

Drawing:

Area (number and units):

Work:



$L=2, W=4$ . Area is equal  $L \cdot W = 2 \cdot 4 = 8 \text{ inch}^2$

**Exercise 8b** (6 points) A square is a rectangle whose length equal to its width. Let the side of a given square be denoted by  $S$ . Write a formula expressing the area of this square in terms of  $S$ . For full credit justify your answer.

Answer:  $S^2$

Justification:

A square is a rectangle for which  $L = W = S$ . Therefore the area of the square with side  $S$  is equal to  $S$  times  $S$ , that is  $S^2$ .

**Exercise 8c** (6 points) A cube is a geometric solid all of whose faces are squares. Let each face of a given cube be a square with side equal to 2.2 inches. Draw the cube and calculate its entire surface area. Specify units. For full credit show your work.

Drawing:

Answer:

Work:

Each face is a square with side equal to 2.2 inches. By **b** the area of the face is equal to

$(2.2)^2 = 4.84$ . The cube has 6 faces: top, bottom, and the four sides.

All the faces are the same. Therefore the entire surface area is equal to  $6 \cdot 4.84 = 29.04$ .



**Exercise 9** Solve the following equations for the specified variable. For full credit show your work.

**Exercise 9a** (6 points) Solve for x.  $\frac{2x}{5} - 3(x + 2) = 1 - x$

Work:

$$\begin{aligned} \frac{2x}{5} - 3(x + 2) &= 1 - x \rightarrow 2x - 15(x + 2) = 5(1 - x) \rightarrow 2x - 15x - 30 = 5 - 5x \rightarrow \\ -13x - 30 &= 5 - 5x \rightarrow -8x = 35 \rightarrow x = -\frac{35}{8} \end{aligned}$$

Answer:  $x = -\frac{35}{8}$

**Exercise 9b** (6 points) Solve for x.  $5 = \frac{Ay}{A - x}$

Work:

$$\begin{aligned} 5 &= \frac{Ay}{A - x} \rightarrow \frac{5}{1} = \frac{Ay}{A - x} \rightarrow 5 \cdot (A - x) = 1 \cdot Ay \rightarrow \\ 5A - 5x &= Ay \rightarrow 5A - Ay = 5x \rightarrow x = \frac{5A - Ay}{5} \end{aligned}$$

**Exercise 9c** (6 points) Solve for x.  $2 + |5x - 7| = 6$

$$2 + |5x - 7| = 6 \rightarrow |5x - 7| = 6 - 2 \rightarrow |5x - 7| = 4$$

This implies that  $5x - 7 = 4$  or  $5x - 7 = -4$ . We solve each equation separately.

$$5x - 7 = 4 \rightarrow 5x = 11 \rightarrow x = \frac{11}{5}. \text{ The other answer is: } x = \frac{11}{5}, \quad x = \frac{3}{5}$$

$$5x - 7 = -4 \rightarrow 5x = 3 \rightarrow x = \frac{3}{5}$$

**Exercise 10** (6 points) Solve the following inequality. For full credit show your work.

$$\frac{2}{3}(x + 5) \leq 3x + 10$$

$$\frac{2}{3}(x + 5) \leq 3x + 10 \rightarrow 2x + 10 \leq 9x + 30 \rightarrow -7x \leq 20 \rightarrow x \geq -\frac{20}{7}$$

**Good Luck!**