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ON THE WEAK DIMENSION OF COHERENT GROUP RINGS

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Let R be a commutative ring with identity, let G be a multiplicative abelian group and denote by RG the group ring of G over R. The properties of RG as reflected by both R and G are of considerable interest.

In (4), Connell proved that RG is a Noetherian ring if and only if R is a Noetherian ring and G is a finitely generated group. Motivated by this result we ask when is RG a coherent ring. Theorem 1 provides an answer to this question.

We next turn our attention to the weak dimension of RG when RG is a coherent ring. A characterization of group rings of weak dimension zero, that is, absolutely flat (or Von Neumann regular) rings, has been known for quite some time and proved independently

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by Auslander (1), McLaughlin (11) and Villamayor (16). These rings are automatically coherent.

In Proposition 1, we generalize a result of Auslander to abelian groups of arbitrary finite rank with coherent group rings. This generalization allows us to completely characterize coherent group rings of finite weak dimension. This is done in Theorem 3. As corollaries we obtain yet another proof of the characterization of absolutely flat group rings, a complete characterization of semihereditary group rings and, comparing Corollary 3 with the result in that direction proved by Gilmer and Parker (5), we conclude that for rings of the form $R[x,x^{-1}]$, the semihereditary condition, and the Prüfer ring condition coincide.

We then investigate to what extent the conditions involved in the characterization of coherent group rings of finite weak dimension characterize coherent regular group rings. This is done in Theorem 4 and Remark 1.

We first determine when is a group ring coherent. To this end we start by proving the following lemma.

LEMMA 1. Let R be a commutative ring and let $x_1, x_2, \ldots, x_n$ be indeterminates over R. Then:

(1) $R[x_1 + x_1^{-1}, \ldots, x_n + x_n^{-1}] \cong R[x_1, \ldots, x_n]$.

(2) $R[x_1, x_1^{-1}, \ldots, x_n, x_n^{-1}]$ is a finitely generated and free $R[x_1 + x_1^{-1}, \ldots, x_n + x_n^{-1}]$ module.

Proof: We prove both statements by induction on the number of variables, n.

(1) For $n = 1$ set $x = x_1$ and let $a_k(x + x^{-1})^k + . . . + a_1(x + x^{-1}) + a_0 = 0$ with $a_i \in R$ for $0 \leq i \leq k$. Multiply this equality by x^k to obtain: $a_k(x^2 + 1)^k + . . . + a_1(x^2 + 1)x^{k-1} + a_0x^k = 0$. This equality holds if and only if $a_k = . . . = a_1 = a_0 = 0$; therefore, $R[x + x^{-1}] \cong R[x]$.

By the induction hypothesis we have that $R[x_1 + x_1^{-1}, . . ., x_n + x_n^{-1}] \cong R[x_1, . . ., x_{n-1}][x_n + x_n^{-1}]$. By the case for $n = 1$ this last ring is isomorphic to $R[x_1, . . ., x_n]$ and the claim is complete.

(2) For $n = 1$ set $x = x_1$. We have $x^{2i} - (x + x^{-1})x^i + 1 = 0$ for $i = 1$ or -1; therefore, $R[x,x^{-1}] = R[x + x^{-1}, x]$ is generated by 1 and x as an $R[x + x^{-1}]$ module. Consider the $R[x + x^{-1}]$-homomorphism $\rho:R[x + x^{-1}] \oplus R[x + x^{-1}] \to R[x,x^{-1}]$ defined by $\rho((f,g)) = f + gx$ for $f,g \in R[x + x^{-1}]$. ρ is onto and our claim for $n = 1$ will be complete if we show that $\ker \rho = 0$. Let $(f,g) \in \ker \rho$. Write $f = a_0 + a_1(x + x^{-1}) + . . . + a_k(x + x^{-1})^k$ and $g = b_0 + b_1(x + x^{-1}) + . . . + b_k(x + x^{-1})^k$ with $a_i,b_j \in R$, $0 \leq i, j \leq k$. Then $f + gx = 0$ implies that $fx^k + gx^{k+1} = 0$ and we obtain the following equality:

$$a_0x^k + a_1(x^2 + 1)x^{k-1} + . . . + a_k(x^2 + 1)^k =$$
$$-(b_0x^{k+1} + b_1(x^2 + 1)x^k + . . . + b_k(x^2 + 1)^kx).$$

Comparing coefficients of powers of x on both sides we obtain:

$b_k = 0$, $a_k = 0$, $b_{k-1} = 0$, $a_{k-1} = 0$, . . ., $b_0 = 0$, $a_0 = 0$, and conclude that $(f,g) = 0$.

By the case for $n = 1$ we have that $R[x_1, x_1^{-1}, \ldots, x_n, x_n^{-1}]$ is a finitely generated free $R[x_1 + x_1^{-1}][x_2, x_1^{-1}, \ldots, x_n, x_n^{-1}]$ module. By the induction hypothesis this last ring is a finitely generated free $R[x_1 + x_1^{-1}, \ldots, x_n + x_n^{-1}]$ module. It follows that $R[x_1,x_1^{-1}, \ldots, x_n,x_n^{-1}]$ is a finitely generated and free $R[x_1 + x_1^{-1}, \ldots, x_n + x_n^{-1}]$ module and the proof is complete.

Recall that a ring R is called *a coherent ring* if every finitely generated ideal of R is finitely presented. A ring R is called *a stably coherent ring* if R is a coherent ring and if for every positive integer n the polynomial ring in n variables over R is a coherent ring.

THEOREM 1. *Let R be a commutative ring and let G be an abelian group, then:*

(1) If G is a torsion group, then RG is a coherent ring if and only if R is a coherent ring.

(2) If $0 < \text{rank } G = n < \infty$, then RG is a coherent ring if and only if the polynomial ring in n variables over R is a coherent ring.

(3) If rank $G = \infty$, then RG is a coherent ring if and only if R is a stably coherent ring.

Proof: Necessary conditions for the coherence of RG:

Assume that RG is a coherent ring. If H is a subgroup of G, then RG is a free, hence faithfully flat, RH module (9, Chapter 2, 2.15). It follows from (7, Corollary 2.1) that RH is a coherent ring.

(1) For $H = \{1\}$ the above remark implies that R is a coherent ring.

(2) If $0 < \text{rank } G = n < \infty$, let H be a free subgroup of G of rank n, then RH is a coherent ring and $RH \simeq R[x_1, x_1^{-1}, \ldots, x_n, x_n^{-1}]$ (9, Chapter 2, 2.6). By Lemma 1 RH is a finitely generated free, hence a finitely presented, module over a polynomial ring in n variables over R. Thus, the polynomial ring in n variables over R is a coherent ring (7, Corollary 2.1) or (14, Proposition 1.1).

(3) If rank $G = \infty$ then for every positive integer n, G contains a free subgroup of rank n. By the above argument the polynomial ring in n variables over R is a coherent ring; therefore, R is a stably coherent ring.

Sufficient conditions for the coherence of RG:

Write $G = \varinjlim G_\alpha$ where $\{G_\alpha\}$ is the set of all finitely generated subgroups of G ordered by inclusion. If $G_\alpha \subset G_\beta$ then RG_β is a free RG_α module and $RG = \varinjlim RG_\alpha$. Therefore, to obtain that RG is a coherent ring, it suffices to show that RG_α is coherent for every α(3, Chapter 1, exercise 12(e), §2).

(1) If G is a torsion group, for each α, G_α is a finite group and so RG_α is a finitely generated free module over the coherent ring R. It follows that RG_α is a coherent ring (7, Corollary 1.2) for every α, and, thus, RG is a coherent ring.

(2) If $0 < \text{rank } G = n < \infty$ then rank $G_\alpha \leq n$ for every α. Since $R[x_1, \ldots, x_n]$ is a coherent ring we have that $R[x_1, \ldots, x_i]$ is a coherent ring for every $1 \leq i \leq n$. Write $G_\alpha = G'_\alpha \times F_\alpha$, where G'_α is the torsion subgroup of G_α and F_α is free of rank equal to $i = \text{rank } G_\alpha$. Then $RG_\alpha = (RF_\alpha)G'_\alpha$ (9, Chapter 2, 2.16). Since G'_α is a torsion group, it follows from the previous case that to obtain that RG_α is a coherent ring, it suffices to show that RF_α is a coherent ring. Now $RF_\alpha \simeq R[x_1, x_1^{-1}, \ldots, x_i, x_i^{-1}]$ and is, therefore, a finitely generated free module over a polynomial ring in i variables over R. Since this polynomial ring is a coherent ring so is RF_α and the claim is complete.

(3) If rank $G = \infty$ we still have rank $G_\alpha < \infty$ for every α. The stable coherence of R implies that RG_α is a coherent ring for every α, and RG is a coherent ring.

As a result of this theorem we obtain a large class of rings R for which RG is a coherent ring for every abelian group G, namely all stably coherent rings. To name a few: Noetherian rings, absolutely flat rings (18), semihereditary rings (18), coherent rings of global dimension two (18), coherent rings of finitely presented dimension two, as defined by Ng (13).

We next turn our attention to the weak dimension of a coherent group ring. We will freely make use of several properties of the group ring which we briefly describe here.

Let G be an abelian group and let H be a subgroup of G. Since RG is a free RH module we have that for every RG module M, $w.\dim_{RH} M \leq w.\dim_{RG} M$ (1). In fact, RG considered as an RH module contains RH as a direct summand and, therefore, $w.\dim RH \leq w.\dim RG$ (1). We also have that if $G = \varinjlim G_\alpha$, where $\{G_\alpha\}$ is a set of subgroups of G ordered by inclusion, then, for any RG module M, $w.\dim_{RG} M = \sup_\alpha \{w.\dim_{RG_\alpha} M\}$ and $w.\dim RG = \sup_\alpha \{w.\dim RG_\alpha\}$ (1).

The <u>augmentation map</u> aug:RG $\to$ R is defined as follows. Let $x = \sum_{g \in G} x_g g \in RG$, where $x_g \in R$ and all but finitely many x_g are zero, then $aug(x) = \sum_{g \in G} x_g$. Via this map, R becomes an RG module. If $r \in R$ and $x = \sum_{g \in G} x_g g \in RG$ then we define $rx = r\, aug(x)$. Whenever we refer to R as an RG module we shall mean that R is the RG module with scalar multiplication defined above. Let $I(G) = \ker(aug)$, then $0 \to I(G) \to RG \to R \to 0$ is an exact sequence of RG modules and I(G) is the ideal of RG generated by all elements of the form $g - 1$, where g runs over a generating set for G. In general, if H is a subgroup of G, then RH is naturally included in RG. The extension of the ideal of RH, I(H) to RG, denoted by I(H)RG, is generated, as an ideal of RG, by all elements of the form h - 1, where h runs over a generating set for H (9, Chapter 3, 1.3), and R(G/H) ≅ RG/I(H)RG (9, Chapter 2, 2.10).

CHANGE OF RINGS THEOREM (8, Theorem 3, p. 172). Let A be a ring, let x be a nonzero divisor in A and let M be an A/xA module, then if $\text{proj.dim}_{A/xA}M = n < \infty$ we have that $\text{proj.dim}_A M = n + 1$.

Let G be an abelian group and let R be a commutative ring which is uniquely divisible by the order of every element of G (of finite order). This condition, which will be extensively used in this paper, is equivalent to the following condition: If $g \in G$ with order $o(g) = p$, where p is prime, then p is a unit in R.

PROPOSITION 1. Let G be an abelian group of finite rank and let R be a commutative ring satisfying that RG is a coherent ring. Then the following conditions are equivalent:

(1) $\text{w.dim}_{RG}R = \text{rank } G$.

(2) $\text{w.dim}_{RG}R < \infty$.

(3) R is uniquely divisible by the order of every element of G.

Proof: We will prove all implications by induction on n = rank G. The case n = 0 was proved by Auslander (1, Proposition 6); thus, we may assume that $n \geq 1$.

(1) $\rightarrow$ (2) Clear.

(2) $\rightarrow$ (3) If H is a subgroup of G then $w.\dim_{RH}R \leq w.\dim_{RG}R < \infty$. Pick H such that rank H < rank G and H contains the torsion subgroup of G. Then the induction hypothesis implies that (3) is satisfied.

(3) $\rightarrow$ (1) Assume first that G is a finitely generated group. Write $G = H \times C$, where C is infinite cyclic. $H \cong G/C$ and write $C = \langle c \rangle$. rank H = n - 1, RG is a free RH module; thus, RH is a coherent ring (7, Corollary 2.1). Moreover, as G (respectively H) is a finitely generated group, I(G) (respectively I(H)) is a finitely generated ideal of RG (respectively RH). It follows that R is a finitely presented RG (respectively RH) module. By (12, Lemma 1.2) for L = G or H we have $w.\dim_{RL}R = \text{proj}.\dim_{RL}R$. As H satisfies (3), by the induction hypothesis we obtain $w.\dim_{RH}R = \text{rank } H$. Now $RH = R(G/C) \cong RG/I(C)RG \cong RG/(c - 1)RG$, and $(c - 1)$ is a nonzero divisor on RG (9, Chapter 2, 2.18). By the change of rings theorem $w.\dim_{RG}R = \text{rank } H + 1 = \text{rank } G$.

If G is not finitely generated write $G = \varinjlim G_\alpha$, where $\{G_\alpha\}$ is the set of all finitely generated subgroups of G ordered by inclusion. Then rank $G_\alpha \leq$ rank G for every α, and there exists an α_0 such that rank G_{α_0} = rank G. We conclude that $w.\dim_{RG}R = \sup_\alpha \{w.\dim_{RG_\alpha}R\} = \sup_\alpha \{\text{rank } G_\alpha\} = \text{rank } G$.

DIMENSION INEQUALITY THEOREM (1, Proposition 4). Let R be a commutative ring and let G be an abelian group. Then: $w.\dim R \leq w.\dim RG \leq w.\dim R + w.\dim_{RG} R$.

THEOREM 2. Let G be an abelian group and let R be a commutative ring which is uniquely divisible by the order of every element of G. If RG is a coherent ring, then $w.\dim RG = w.\dim R + \operatorname{rank} G$.

Proof: If $w.\dim R = \infty$ then $w.\dim RG \geq w.\dim R = \infty$ and equality holds. We may, therefore, assume that $w.\dim R < \infty$.

Assume first that G is finitely generated. We will prove the theorem in this case by induction on $n = \operatorname{rank} G$. The case for $n = 0$ was proved by Auslander (1, Theorem 7). The transition from the $n - 1$ to the n step is done in the same set up as in Proposition 1, (3) $\rightarrow$ (1). By the induction hypothesis we have $w.\dim RH = w.\dim R + \operatorname{rank} H$. By Proposition 1 we have that $w.\dim_{RG} R = \operatorname{rank} G$ and, therefore, $w.\dim RG \leq w.\dim R + \operatorname{rank} G$.

As RG (respectively RH) is a coherent ring, for L = G or H we have that $w.\dim RL = \sup \{w.\dim_{RL} M \mid M$ is a finitely presented RL module$\}$ (12, Proposition 1.1), and $w.\dim_{RL} M = \operatorname{proj.dim}_{RL} M$ if M is a finitely presented RL module (12, Lemma 1.2). Let M be a finitely presented RH module with $w.\dim_{RH} M = w.\dim R + \operatorname{rank} H$. then M is a finitely presented RG module and, by the change of rings theorem, we have $w.\dim_{RG} M = w.\dim R + \operatorname{rank} G$. We conclude that $w.\dim RG \geq w.\dim R + \operatorname{rank} G$ and equality holds.

If G is not finitely generated and if rank $G = \infty$, then for every positive integer n, G contains a free subgroup F_n of rank $F_n = n$. As RF_n is a coherent ring we have $w.\dim RG \geq w.\dim RF_n = w.\dim R + n$ for all n, and the equality of the theorem holds.

We may, therefore, assume that rank $G < \infty$. Write $G = \varinjlim G_\alpha$, where $\{G_\alpha\}$ is the set of all finitely generated subgroups of G ordered by inclusion. Rank $G_\alpha \leq$ rank G for every α, and there exists an α_0 such that rank G_{α_0} = rank G. Now $w.\dim RG = \sup_\alpha \{w.\dim RG_\alpha\} = \sup_\alpha \{w.\dim R + \text{rank } G_\alpha\} = w.\dim R + \text{rank } G$.

THEOREM 3. <u>Let R be a commutative ring and let G be an abelian group satisfying that RG is a coherent ring. Then $w.\dim RG < \infty$ if and only if there exists a nonnegative integer n such that R, G and n satisfy: $w.\dim R \leq n$, rank $G = n - w.\dim R$ and R is uniquely divisible by the order of every element of G.</u>

<u>If such an integer n exists then $w.\dim RG = n$.</u>

<u>Proof</u>: If such an integer n exists then Theorem 2 yields $w.\dim RG = n < \infty$. Conversely, assume that $w.\dim RG < \infty$. Write $w.\dim RG = n$. Then $w.\dim R \leq w.\dim RG = n$. Let $g \in G$ with order $o(g) = p$, where p is prime, then g is contained in a finitely generated subgroup H of G. As $w.\dim_{RH} R \leq w.\dim_{RG} R \leq n$ by Proposition 1, p is a unit in R, and R is uniquely divisible by the order of every element of G. It now follows from Theorem 2 that rank $G = n - w.\dim R$.

As absolutely flat rings are stably coherent rings we obtain:

COROLLARY 1. Let R be a commutative ring and let G be an abelian group, then RG is an absolutely flat ring if and only if R is an absolutely flat ring which is uniquely divisible by the order of every element of G, and G is a torsion group.

Recall that a ring R is called a semihereditary ring if every finitely generated ideal of R is projective. A ring R is a semihereditary ring if and only if R is a coherent ring with w.dim R $\leq$ 1 (12, Proposition 2.2). As semihereditary rings are stably coherent rings, combining Theorem 2 and Theorem 3 we obtain:

COROLLARY 2. Let R be a commutative ring and let G be an abelian group, then RG is a semihereditary ring if and only if exactly one of the following conditions holds:

(1) R is an absolutely flat ring which is uniquely divisible by the order of every element of G and rank G $\leq$ 1.

(2) R is a semihereditary (not absolutely flat) ring which is uniquely divisible by the order of every element of G and G is a torsion group.

As a corollary of Corollary 2, we also obtain:

COROLLARY 3. Let R be a commutative ring and let x be an indeterminate over R. Then $R[x,x^{-1}]$ is a semihereditary ring if and only if R is an absolutely flat ring.

Proof: Note that $R[x,x^{-1}] = RG$, where G is an infinite cyclic group (9, Chapter 2, 2.6).

In (5), Gilmer and Parker defined a Prüfer ring to be a ring where every finitely generated regular ideal is invertible. As invertible ideals are projective, this condition on a ring is a restricted form of semihereditarity. In view of Corollary 3 and Proposition 2.5 of (5), we see that for a ring of the form $R[x,x^{-1}]$, the semihereditary condition and the Prüfer ring condition coincide.

A condition closely related to the finiteness of the weak dimension is that of regularity. A ring R is called a regular ring if every finitely generated ideal of R has finite projective dimension. This notion, which has been extensively studied for Noetherian rings, was extended to coherent rings with a considerable degree of success (2), (15), (17).

If R is a coherent regular ring and M is a finitely presented R module, one can prove by induction on the number of generators of M, that $\text{proj.dim}_R M < \infty$ (15).

It is clear that a coherent ring of finite weak dimension is a regular ring. The converse is not necessarily true. For example, the ring $R = k[x_1, x_2, \ldots]$, where k is a field and $x_1, x_2, \ldots$ are indeterminates over k, is a coherent regular ring of infinite weak dimension. To see this, note that $R = \varinjlim k[x_1, x_2, \ldots, x_n]$. $w.\dim k[x_1, \ldots, x_n] = n$; thus, $k[x_1, \ldots, x_n]$ is a Noetherian regular ring for all n. R is a coherent regular ring by (10, Proposition 1.36). As R is a faithfully flat $k[x_1, \ldots, x_n]$ module for every n, we have that $w.\dim R \geq n$ for every n (10, Proposition 1.34).

LEMMA 2. Let $R \subset S$ be commutative rings such that S is a faithfully flat R module. If S is a coherent regular ring, then so is R.

Proof: As S is a faithfully flat R module, we have that R is a coherent ring (7, Corollary 2.1). Let I be a finitely generated ideal of R, then I is a finitely presented R module and so $I \otimes_R S$ is a finitely presented S module. Let $\text{proj.dim}_S I \otimes_R S = n < \infty$. We conclude that if N is any R module, then $\text{Ext}_S^k(I \otimes_R S, N \otimes_R S) = 0$ for $k > n$. By (10, Lemma 1.32) $0 = \text{Ext}_S^k(I \otimes_R S, N \otimes_R S) = \text{Ext}_R^k(I,N) \otimes_R S$ for every $k > 0$. Since S is a faithfully flat R module we have that $\text{Ext}_R^k(I,N) = 0$ for $k > n$. We conclude that $\text{proj.dim}_R I \leq n$, and, therefore, R is a coherent regular ring.

We explore to what extent the conditions involved in the characterization of coherent group rings of finite weak dimension characterize coherent regular group rings. We make the following conjecture:

Let R be a commutative ring and let G be an abelian group such that the group ring RG is a coherent ring. Then RG is a regular ring if and only if R is a coherent regular ring which is uniquely divisible by the order of every element of G.

The next theorem shows that the conditions of the conjecture are necessary, and provides two cases in which the conditions are sufficient as well.

THEOREM 4. Let R be a commutative ring and let G be an abelian group such that the group ring RG is a coherent ring. If RG is a regular ring then R is a coherent regular ring which is uniquely divisible by the order of every element of G. Conversely, if R is a coherent regular ring which is uniquely divisble by the order of every element of G, and, in addition, either G is a torsion group or w.dim R $< \infty$, then RG is a regular ring.

Proof: Assume that RG is a regular ring; then by Lemma 2, R is a coherent regular ring. Let $g \in G$ with $o(g) = p$, where p is prime; then $g \in H$, where H is a finitely generated subgroup of G. By Lemma 2, RH is a coherent regular ring as well. Since H is finitely generated, R is a finitely presented RH module and, thus, $w.\dim_{RH}R = \text{proj.}\dim_{RH}R < \infty$. By Proposition 1, this implies that p is a unit in R, and, thus, R is uniquely divisible by the order of every element of G.

Conversely, write $G = \varinjlim G_\alpha$, where $\{G_\alpha\}$ is the set of all finitely generated subgroups of G ordered by inclusion. Then $RG = \varinjlim RG_\alpha$, and, by (10, Proposition 1.36), to show that RG is a regular ring, it suffices to show that for each α, RG_α is a regular ring. Since RG is a coherent ring, so is RG_α for every α. We may, therefore, assume that G is a finitely generated group.

If G is a torsion group, then G is a finite group and the order of G, o(G) is a unit in R. Let I be a finitely generated ideal of RG, then I is a finitely presented RG module. Since RG is a finitely generated free R module, we have that I is a finitely presented R module. Let $n = \text{proj.}\dim_R I < \infty$ and let $0 \to X_n \to F_{n-1} \to \ldots \to F_0 \to I \to 0$ be an exact sequence of RG modules with F_i free RG modules. Since F_i are free as R modules as well, we have that X_n is a projective R module. But then, by (9, Chapter 3, 1.4), X_n is a projective RG module and $\text{proj.}\dim_{RG} I \leq n$. We conclude that RG is a regular ring.

If $w.\dim R < \infty$, by Theorem 2 we have $w.\dim RG < \infty$; therefore, RG is a regular ring.

REMARK 1. While this paper was in the reviewing process the author had solved the above conjecture affirmatively. Its proof, given in (6, Theorem 2.7), is based on the following result proved by us in (6, Proposition 2.5).

RESULT. Let R be a coherent regular ring and let I be an ideal of the polynomial ring in one variable over R, R[x], admitting an exact sequence $\cdots \to F_1 \to F_0 \to I \to 0$, with F_i finitely generated and free R[x] modules. Then $\text{proj.dim}_{R[x]} I < \infty$.

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REFERENCES

1. M. Auslander. On regular group rings. Proc. A.M.S. 8 (1957), 658-664.

2. J. Bertin. Anneaux cohérents reguliers. C.R. Acad. Sc. Paris, Ser A, 273 (1971), 1-2.

3. N. Bourbaki. Commutative algebra. Addison-Wesley (1972).

4. I. G. Connell. On the group ring. Canad. J. Math. 15 (1963), 650-685.

5. R. Gilmer & T. Parker. Semigroup rings as Prüfer rings. Duke Math J., 41 (1974), 219-230.

6. S. Glaz. Regular symmetric algebras. To appear in J. of Alg.

7. M. E. Harris. Some results on coherent rings. Proc. A.M.S. 17 (1966), 474-479.

8. I. Kaplansky. Fields and rings. The Univ. of Chicago Press (1969).

9. G. Karpilovsky. Commutative group algebras. Marcel Dekker Lecture Notes 78 (1983).

10. K. P. McDowell. Commutative coherent rings. McMaster University, Math. Report 66 (1974).

11. J. E. McLaughlin. A note on regular group rings. Mich. Math. J. 5 (1958), 127-128.

12. D. G. McRae, Homological dimensions of finitely presented modules. Math. Scand. 28 (1971), 70-76.

13. H. K. Ng. Coherent rings of dimension two. Proc. A.M.S. 91 (1984), 518-521.

14. Y. Quentel. Sur l'uniforme cohérence des anneaux noethériens. C.R. Acad. Sc. Paris 275 (1972), 753-755.

15. Y. Quentel. Sur le theoreme d'Auslander-Buchsbaum. C.R. Acad. Sc. Paris Ser A, 273 (1971), 880-881.

16. E. Villamayor. On weak dimensions of algebras. Pacific J. Math. 9 (1959), 941-951.

17. W. V. Vasconcelos. Divisor theory in module categories. North. Holland Math. Studies 14 (1974),

18. W. V. Vasconcelos. The rings of dimension two. Marcel Dekker Lecture Notes 22 (1976).

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