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FIXED RINGS OF COHERENT REGULAR RINGS

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1. INTRODUCTION.

Let R be a commutative ring, let G be a group of automorphisms of R and denote by R^G the fixed subring of R. $R^G = \{ x \in R / g(x) = x$ for all $g \in G \}$.

In this paper we explore the conditions under which the finiteness property of coherence and several other related homological properties of R are inherited by R^G. The paper is divided into three sections.

Section 2 investigates descent of coherence from R to R^G. This question has its roots in the classical investigations of descent of finiteness from R to R^G carried out by Zariski and Rees--to partially solve Hilbert's fourteenth problem [22], and by Emmy Noether--to determine when is R^G an affine algebra [1]. More recent and more closely related to the question of coherence is the determination of conditions under which we have descent of Noetherianess. This question was explored in both the commutative and the noncommutative settings by several authors [2, 4, 6, 19, 20, 21]. In this section we determine conditions under which descent of coherence is possible and construct several examples that show that the restrictions imposed on R and on G to insure descent of coherence cannot be relaxed.

Section 3 and Section 4 deal with aspects of descent of coherent regularity from R to R^G. This kind of investigation is part of a general pattern of exploration of coherent regularity [8, 10], and is also related to recent investigations into the relations between projectivity over R and over R^G [15, 17]. More particularly:

Section 3 concentrates on descent of coherent regularity in the classical cases, that is in the cases where R is a coherent ring of w.dim R $\leq$ 1. These cases cover Von Neumann regular, and semihereditary rings. The appropriate conditions necessary for descent in these cases were already known [2, 3, 14]. We expose the underlying reason for the ease of descent in these cases, namely, the prevailing existence of faithful flatness. In this section we also explore several instances where R is actually a projective R^G module.

Section 4 considers general descent of coherent regularity. It is shown, by an example, that for rings R with w.dim R $\geq$ 2, the conditions which insured descent of coherent regularity for smaller weak dimensions no longer suffice. We then impose several stronger restrictions on R and on G under which general descent of coherent regularity is still possible.

2. COHERENCE.

Let A and B be two rings. A is called a <u>module retract</u> of B if $A \subset B$ and there exists an A-module homomorphism $\varphi : B \longrightarrow A$ satisfying $\varphi(a) = a$ for every element a of A. If such a map φ exists it is called a <u>module retraction map</u>.

Note that if A is a module retract of B, then the module retraction map φ splits the identity map of A, thus B contains A as an A module direct summand. It follows that A is a pure A submodule of B and that no proper ideal of A blows up in B.

In [2] Bergman points out the existence of module retraction maps $\varphi : R \to R^G$ in several cases:

(1) G is a finite group and o(G), the order of G, is a unit in R. In this case for every $x \in R$ $\varphi(x) = \frac{1}{o(G)} \sum_{g \in G} g(x)$..

(2) G is a locally finite group (that is for every $x \in R$ the orbit of x, Gx has finite cardinality $n(x) < \infty$), and n(x) is a unit in R for every x in R. In this case for every $x \in R$,

$$\varphi(x) = \frac{1}{n(x)} \sum_{y \in Gx} y.$$

Bergman proves [2] that if R^G is a module retract of R and R is a Noetherian ring then so is R^G. That this result cannot be much improved one can see by considering the example of a finite group and a Noetherian ring R with no o(G)-torsion which fails to descent Noetherianess [4, 21], and the example of an infinite cyclic group and a P.I.D. R which fails to descent Noetherianess [2].

In case R is a coherent ring it is necessary to impose stronger restrictions than the existence of a module retraction map from R to R^G.

THEOREM 1. Let R be a coherent ring, and let G be a group of automorphisms of R, then R^G is a coherent ring in the following cases:

(1) G is a locally finite group or R^G is a module retract of R and R is a flat R^G module.

(2) <u>R^G is a module retract of R and R is a finitely generated R^G module.</u>

<u>Proof</u>: (1) If G is a locally finite group R is an integral extension of R^G [21], and thus in both cases R is a faithfully flat R^G module. It follows that R^G is coherent along with R [9, Corollary 2.4.5].

(2) Let $\varphi : R \longrightarrow R^G$ be a module retraction map. Let R^n be a free R module with basis $e_1,\dots,e_n$, and let $(R^G)^n$ be a free R^G module with basis $e'_1,\dots,e'_n$. Then φ can be extended to an R^G module homomorphism, also denoted by φ, $\varphi : R^n \longrightarrow (R^G)^n$, where for $r_i \in R$ $\varphi(\sum_{i=1}^{n} r_i e_i) = \sum_{i=1}^{n} \varphi(r_i)e'_i$. Let $I = (a_1,\dots,a_n)$ be a finitely generated ideal of R^G and let $0 \longrightarrow K \longrightarrow R^n \xrightarrow{f} IR \longrightarrow 0$ be an exact sequence of R modules with $f(e_i) = a_i$, $1 \le i \le n$. Then $K = \ker f$ is a finitely generated R, and hence R^G, module. If the sequence $0 \longrightarrow L \longrightarrow (R^G)^n \xrightarrow{f'} I \longrightarrow 0$ with $f'(e'_i) = a_i$ $1 \le i \le n$, is an exact sequence of R^G modules, then φ maps K onto L. It follows that I is finitely presented and so R^G is a coherent ring.

We now construct a series of examples which show that the restrictions imposed on R and on G in Theorem 1 cannot be relaxed.

EXAMPLE 2. <u>Finite groups</u>.

Let k be a field of $\operatorname{ch} k \neq 2$, let x,y and $\{z_i\}_{i=1}^{\infty}$ be indeterminates over k, and let $R = k[x,y,z_i, i \ge 1]$. R is a coherent ring.

Let g be an automorphism of R defined by: $g(\alpha) = \alpha$ for all $\alpha \in k$, $g(x) = -x$, $g(y) = -y$, $g(z_i) = -z_i$, for $i \ge 1$, and let $G = \langle g \rangle$, then $o(G) = 2$ is a unit in R, and so R^G is a module retract of R.

$R^G = k[\, x^2, y^2, xy, z_i^2, xz_i, yz_i,\ i \geq 1,\,]$ is not a coherent ring since the ideal $(\, xy : x^2\,) = (\, y^2, xy, yz_i,\ i \geq 1,\,)$ is not a finitely generated ideal of R^G.

A somewhat weaker example is provided by the well known Nagarajan's example [21]. We sketch it here. Let $R = F[[\, x,y\,]]$ be the power series ring in indeterminates x and y over the field $F = Z_2(\, a_i, b_i,\ i \geq 1\,)$, where Z_2 is the prime field of characteristic 2, and $a_i, b_i,\ i \geq 1$ are infinitely many indeterminates. Set $p_i = a_i x + b_i y$, and define an automorphism of R,g, by $g(x) = x$, $g(y) = y$, $g(a_i) = a_i + yp_{i+1}$, $g(b_i) = b_i + xp_{i+1}$, $i \geq 1$. Let $G = <g>$, then $O(G) = 2$, but two, of course, is not a unit in R. In [21] it is proved that R^G is not a Noetherian ring. William Heinzer pointed out that R^G is not a coherent ring as well. To see this he provided the following argument: R^G is a strongly Laskerian ring [11]. Let P be a prime ideal of R^G. Since R^G, as a Laskerian ring, has a Noetherian spectrum, P is the radical of a finitely generated ideal, say I. Since R^G is strongly Laskerian $P = I : a$ for some element a of R^G. If R^G were a coherent ring, this will imply that P itself is finitely generated and, hence, that R^G is Noetherian.

EXAMPLE 3. Infinite groups

The constructions described here are based on a similar construction of Bergman [2], with the idea going back to Gilmer [7].

Let A be an integrally closed domain with field of quotients K. For a valuation v of K denote by $K_v = \{\, x \in K,\ v(x) \geq 0\, \}$ the valuation ring of v. Let $V = \{\, v,\ v$ valuation on K such that $A \subseteq K_v\, \}$. Since A is integrally closed $A = \cap K_v$.

Let t be an indeterminate over A, v extends to a valuation of A[t] as follows: $v(a_n t^n + \cdots + a_o) = \min_i v(a_i)$. v now extends to a valuation of the field of quotients of A[t], K(t).

Let $R = \cap K(t)_v = \{ P(t)/Q(t), P(t), Q(t) \in A[t], v(P(t)) \geq v(Q(t))$, for all $v \in V \}$. R is a Bezout domain [2] and therefore a coherent ring.

Let B be a subring of A. If $b \in B$, $P(t) \in A[t]$ and $v \in V$ we have $v(P(t+b) \geq v(P(t)$, hence using $-b$, we conclude that $v(P(t+b) = v(P(t))$.

Consider the automorphism $\varphi_b : A[t] \longrightarrow A[t]$, $\varphi_b(t) = t + b$. φ_b can be extended to K(t) and is v invariant, thus φ_b gives an automorphism of R. Let $G = \{ \varphi_b \}_{b \in B}$. If B is infinite then $K(t)^G = K$ and thus $R^G = K(t)^G \cap R = K \cap R = \cap K(t)_v \cap K = A$.

The examples now depend on the choices for A and B. Let k be a field, let x,y be indeterminates over k, and let $A = k[y, x^i y] \subseteq k[y,x]$. A is not a coherent ring since $xyA \cap x^2yA = (x^2y^2, x^3y^2, x^4y^2, \ldots)$ is not a finitely generated ideal of A. A is integrally closed [12, example 3].

If we pick k to be an infinite field of characteristic $p > 0$, and $B = k$, then every element of G has order p. Thus we obtain an example where R is coherent of $ch R = p > 0$. G is an infinite group where each element has finite order and $R^G = A$ is not coherent.

If we pick $B = k$ and $ch k = 0$ no element of G has finite order.

We can modify this construction to obtain an even sharper example.

Let A be a noncoherent Krull domain of characteristic zero. Such a ring is constructed in [5]. More specifically the example constructed in [5] is of a Krull domain of Krull dimension three with a prime ideal P of height one which is not finitely generated. Since a height one prime ideal of a Krull domain is an intersection of two principal fractional ideals, we have that A is not a coherent ring. We now follow the above construction using instead of A[t], the ring $A[\, s,t,s^{-1},t^{-1}\,]$ where s and t are indeterminates, the set V = { v, v essential valuation of A }. Bergman [2] shows that in this case R is actually a P.I.D., and G may be chosen such that G = <g> is an infinite cyclic group. We therefore obtain an example of a Noetherian (P.I.D.) ring R, an infinite cyclic group G and $R^G = A$ is not a coherent ring.

We conclude this section with an amusing case where R descends coherence without necessarily being coherent. Recall that a ring R is called a p.p. ring if every principal ideal of R is projective.

PROPOSITION 4. Let R be a p.p. ring, let G be any group such that gl.dim $R^G \leq 2$, then R^G is a coherent ring.

Proof: Since R is a p.p. ring so is R^G [14]. It follows that the set of minimal primes of R^G, Min(R^G), is compact [9, Theorem 4.2.10]. We now conclude that R^G is a coherent ring by [24, Proposition 6.1].

3. FLATNESS.

A ring A is called a regular ring if every finitely generated ideal of A has finite projective dimension. This notion which has been extensively studied for Noetherian rings has been extended to coherent rings with a considerable degree of success. For a coherent ring A the regularity condition is closely related to the behaviour of the weak dimension of modules over A. In particular a coherent ring of finite weak dimension is a coherent regular ring although the converse is not necessarily true [9, Chapter 6, Section 2]. The classical examples of coherent regular rings are Von Neumann regular rings, that is rings of weak dimension zero, and semihereditary rings, that is precisely those rings which are coherent and of weak dimension less or equal to one [9, Corollary 4.2.19].

Our interest lies in descent of coherent regularity from R to R^G. The cases where the weak dimension of the ring is less or equal to one have been solved by Bergman [2] and Jøndrup [14], as follows:

THEOREM 5 ([2], [14]). (1) Let R be a Von Neumann regular ring and let G be any group of automorphisms of R, then R^G is a Von Neumann regular ring.

(2) Let R be a semihereditary ring and let G be a group of automorphisms of R such that either G is locally finite or R^G is a module retract of R, then R^G is a semihereditary ring.

We remark that both Bergman [2] and Jøndrup [14] proved that if R is a semihereditary ring and G is a finite group then R^G is a semihereditary ring. One extends this result to locally finite groups by noticing that Jøndrup's proof depends only on the existence of finite cardinality orbits for every element of R.

In general we have that if R is a faithfully flat extension of R^G, and R is a coherent ring of finite weak dimension then R^G is a coherent ring of finite weak dimension and w.dim $R^G \leq$ w.dim R [9, Theorem 3.1.1.]. Theorem 7 shows that the underlying reason for the ease of descent of Von Neumann regularity and semihereditarity is the presence of faithful flatness in all cases where descent is known to occur.

LEMMA 6. Let R be a ring and let G be a locally finite group of automorphisms of R. Let a be an element of R with orbit Ga of cardinality n. Denote by $\sigma_1(a), \ldots, \sigma_n(a)$ the elementary symmetric functions of a, that is $\sigma_1(a) = \sum_{y=Ga} y$, $\sigma_2(a) = \sum_{\substack{y,y' \in Ga \\ y \neq y'}} y \cdot y'$, ..., $\sigma_n(a) = \prod_{y \in Ga} y$ then $\mathrm{Ann}_{R^G}(a) \subseteq \bigcap_{i=1}^{n} \mathrm{Ann}_{R^G}(\sigma_i(a)) \subseteq \mathrm{Ann}_{R^G}(a^n)$.

Proof: Let $x \in R^G$ satisfying $xa = 0$, then $xg(a) = 0$ for all $g \in G$ and so $x\sigma_i(a) = 0$ for all i. This proves that $\mathrm{Ann}_{R^G}(a) \subseteq \bigcap_{i=1}^{n} \mathrm{Ann}_{R^G}(\sigma_i(a))$. Now assume that $x \in R^G$ and $x\sigma_i(a) = 0$ for all i, to see that $xa^n = 0$ note that a satisfies the equation $0 = \prod_{y \in Ga} (a-y) = a^n - \sigma_1(a)a^{n-1} + \ldots (-1)^n\sigma_n(a)$.

THEOREM 7. Let R be a ring and let G be a group of automorphisms of R.

(1) If R^G is a Von Neumann regular ring then R is a faithfully flat R^G module.

(2) If R^G is a semihereditary ring, G is a locally finite group and R is a reduced ring then R is a faithfully flat R module.

(3) If R^G is a semihereditary ring and principal ideals of R are flat then R is a flat R^G module.

Proof: (1) In order to show that R is a faithfully flat R^G module it suffices to show that for any proper ideal m of R^G we have $mR \neq R$. Assume the contrary, then $mR = R$ for some ideal m of R implies that there exists a finitely generated ideal $J \subset m$ with $JR = R$. Since R^G is Von

Neumann regular $J = eR^G$ for some idempotent $e \in R^G$, but then $JR = eR = R$ implies $e = 1$, so m is not proper.

(2) Since R is an integral extension of R^G it suffices to show that R is a flat R^G module. Let M be a maximal ideal of R and let $m = R^G \cap M$. We need only show that R_M is a flat R^G_m module. Since R^G is a semihereditary ring we have that R^G_m is a valuation domain, therefore R_M is a flat R^G_m module if and only if R_M is a torsion free R^G_m module.

We now show that R_M is a torsion free R^G_m module. We first note that if $x \in R^G$ is a nonzero divisor in R^G then x is a nonzero divisor in R. For assume that $xa = 0$ for some $a \in R$ then by Lemma 6 $x\,\sigma_i(a) = 0$ for all elementary symmetric functions of a. As $\sigma_i(a) \in R^G$ we have $\sigma_i(a) = 0$, but then $a^n = 0$, and so $a = 0$.

Since R^G_m is a domain it follows that every zero divisor of R^G localizes to zero or to a unit in R^G_m. Now let $x = r/s \in R^G_m$ and $a = u/v \in R_M$ satisfying $xa = 0$, then there exists a $w \in R - M$ such that $wur = 0$ in R. It follows that either $wu = 0$ or r is a zero divisor in R^G. In the first case $a = 0$. In the second case $a = 0$ if $r \notin m$ and $x = 0$ if $r \in m$. Thus R_M is a torsion free R^G_m module.

(3) As in (2) we need only show that if M is a maximal ideal of R, and $m = R^G \cap M$ then R_M is a torsion free R^G_m module. But $R^G_m \subset R_M$ and by [9, Theorem 4.2.2] R_M is a domain. The claim now follows.

In spite of the faithful flatness R might fail to be a projective R^G module even in case R^G is a Von Neumann regular ring [15, Example 5]. Nevertheless there are some cases where this actually happens. Jøndrup [15] proved that if R^G is a semihereditary ring, G is a finite group of automorphisms of R and R is either a reduced ring and a cyclic R^G module, or o(G) is a unit in R and R is a finitely generated R^G module, then R is a projective R^G module. We show that one can considerably relax the conditions on R and on G and still maintain the same conclusion, provided the generators of R over R^G are of a special kind.

THEOREM 8. Let R be a ring, let G be a locally finite group of automorhisms of R and assume that R^G is a p.p. ring, then R is a projective R^G module in the following cases:

(1) R is a reduced ring and a cyclic R^G module.

(2) There exists a finite family of idempotents in R generating R as an R^G algebra.

(3) R^G is semihereditary and R is a reduced ring and a finitely presented R^G module.

Proof: (1) Let $R = aR^G$ for some $a \in R$. It suffices to show that $\mathrm{Ann}_{R^G}(a) = eR^G$ for some idempotent $e \in R^G$. Since R is a reduced ring we conclude by Lemma 6 that $\mathrm{Ann}_{R^G}(a) = \bigcap_{i=1}^{n} \mathrm{Ann}_{R^G}(\sigma_i(a))$, where $\sigma_i(a)$ are the elementary symmetric functions of a. R^G is a p.p. ring therefore $\mathrm{Ann}_{R^G}(\sigma_i(a)) = e_iR^G$ for some idempotents $e_i \in R^G$. Thus $\mathrm{Ann}_{R^G}(a) = eR^G$ for $e = e_1 \cdot \ldots \cdot e_n$.

(2) Let $R = R^G[e_1,\ldots,e_n]$ for a family of idempotents $e_1,\ldots,e_n \in R$. Then R is generated as an R^G module by the following family of idempotents: $\mathcal{E} = \{ e_i, 1 \le i \le n; e_ie_j, i < j, 1 \le i,j \le n; \ldots ; e_1e_2\ldots e_n \}$. Construct a new family of idempotents in R as follows:

$\mathcal{F} = \{ f_i = e_i \prod_{j \neq i} (1 - e_j)$, $1 \leq i \leq n$; $f_{ij} = e_i e_j \prod_{k \neq i,j} (1-e_k)$; $i < j$, $1 \leq i,j \leq n$; ... ;$f_{1...n} = e_1 e_2 ... e_n$ }. One can easily check that the idempotents in $\mathcal{F}$ are mutually orthogonal, that is $f \cdot f' = 0$ for every $f,f' \in \mathcal{F}$, and that $R = R^G[e_1,...,e_n] = \bigoplus_{f \in \mathcal{F}} fR^G$. Therefore to show that R is a projective R^G module we need only show that fR^G is a projective R^G module for every idempotent $f \in R$. Since $f^n = f$ for any positive integer n, we have by Lemma 6 that $\mathrm{Ann}_{R^G}(f)$ is generated by an idempotent and thus fR^G is a projective R^G module.

(3) By Theorem 7 R is a flat R^G module, and since it is finitely presented it is actually projective.

4. REGULARITY.

Let R be a coherent regular ring. If w.dim $R \geq 2$ R^G might fail to be coherent regular even when R^G is a module retract of R, G is finite, both R and R^G are Noetherian rings, and R is a finitely generated R^G module.

EXAMPLE 8. Let k be a field of characteristic zero. Let $R = k[x,y]$ the polynomial ring in two variables x and y over k. R is a Noetherian regular ring of w.dim R = gl.dim R = 2.

Let g be the automorphism $g(\alpha) = \alpha$ for all α in k, $g(x) = -x$, $g(y) = -y$, and let G = <g>, then o(G) = 2 is a unit in R. $R^G = k[x^2,xy,y^2]$ is a Noetherian ring, with w.dim $R^G = \infty$ [6].. Since R^G is a module retract of R we have: gl.dim $R^G \leq$ gl.dim R + proj.dim$_{R^G}$R [16]. It follows that proj.dim$_{R^G}$R $= \infty$. Since R is a finitely generated, hence finitely presented R^G module, this means that R^G is not a regular ring.

We will show in Theorem 10 that what went wrong in this example is precisely the fact that $\text{proj.dim}_{R^G} R = \infty$.

LEMMA 9. Let R be a ring and let G be a group of automorphisms of R such that R^G is a module retract of R and $n = \text{proj.dim}_{R^G} R < \infty$. Let I be an ideal of R^G and assume that $\text{proj.dim}_R IR = m$, then $\text{proj.dim}_{R^G} I \leq n + m$.

Proof: We have that

$\text{proj.dim}_{R^G} IR \leq \text{proj.dim}_R IR + \text{proj.dim}_{R^G} R = n + m$ [15]. Let φ be the module retraction map from R to R^G, then φ maps IR onto I and provides a splitting for the identity map on I. Thus $IR \simeq I \oplus N$ for some R^G module N. Let M be any R^G module then for $s > n + m$

$0 = \text{Ext}^s_{R^G}(I \oplus N, M) = \text{Ext}^s_{R^G}(I, M) \oplus \text{Ext}^s_{R^G}(N, M)$. Therefore $\text{Ext}^s_{R^G}(I, M) = 0$ and $\text{proj.dim}_{R^G} I \leq n + m$.

As a corollary using [9, Corollary 2.5.5] we immediately obtain.

THEOREM 10. Let R be a ring and let G be a group of automorphisms of R such that R^G is a module retract of R and $n = \text{proj.dim}_{R^G} R < \infty$. Then if R is a regular ring so is R^G. If in addition R is a coherent ring of $\text{w.dim}\, R = m < \infty$, then $\text{w.dim}\, R^G \leq n + m$.

COROLLARY 11. Let R be a ring and let G be a group of automorphisms of R such that either R is a faithfully flat R^G module or R^G

is a module retract of R and R is a finitely generated R^G module of finite projective dimension. If R is a coherent regular ring then so is R^G and $w.\dim R^G \leq w.\dim R + \text{proj.dim}_{R^G} R$.

Proof: Use Theorem 10 and [9, Theorem 6.2.5].

We know of only one case where the homological restriction $n = \text{proj.dim}_{R^G} R < \infty$ may be somewhat relaxed. Recall that a ring A is called $\aleph_0$ Noetherian if every ideal of R is countably generated.

COROLLARY 12. Let R be an $\aleph_0$ Noetherian coherent regular ring and let G be a group of automorphisms of R such that R^G is a module retract of R and R is a finitely generated R^G module of finite weak dimension, then R^G is a coherent regular ring and $w.\dim R^G \leq w.\dim R + w.\dim_{R^G} R + 1$.

Proof: We first show that if R is an $\aleph_0$ Noetherian ring and R^G is a module retract of R with retraction map $\varphi : R \longrightarrow R^G$, then R^G is an $\aleph_0$ Noetherian ring. To see this let I be an ideal of R^G. Let $\{a_\alpha\}$, $a_\alpha \in IR$ be a countable generating set of IR as an R module. Let $a_\alpha = \sum_{i=1}^{n_\alpha} u_{\alpha i} r_{\alpha i}$ with $u_{\alpha i} \in I$ and $r_{\alpha i} \in R$. If $a \in I$ then $a = \sum_{\text{finite}} (\sum_{i=1}^{n_\alpha} u_{\alpha i} r_{\alpha i}) s_\alpha$ for some $s_\alpha \in R$. Apply φ to both sides to conclude that $\{ \varphi(u_{\alpha i}) \}$ generate I.

We now apply [23, Lemma 1.1] and [24, Lemma 0.11] to obtain that if R is also $\aleph_0$ generated over R^G then $\text{proj.dim}_{R^G} R \leq \text{w.dim}_{R^G} R + 1$. The result now follows.

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