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★**Commutative coherent rings.**

Lecture Notes in Mathematics, 1371.

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A ring R is said to be coherent if every finitely generated ideal of R is finitely presented; this notion grew out of the important topic of coherent sheaves, coming from algebraic geometry. Thus a Noetherian ring is coherent; however, other examples of such rings are Boolean algebras, absolutely flat rings, valuation and Prüfer rings, and semihereditary rings. So the study of coherent rings contains influences coming from the theory of Noetherian and arithmetic rings, and from homological algebra. This extensive and comprehensive book covers most results in (commutative) coherent ring theory known to date (with a few asides on the noncommutative theory) and very successfully blends the ring-theoretic and homological approaches to the subject. It is designed so as to be suitable for a second-year graduate course, and as a reference book. Containing as it does a wealth of interesting examples and counterexamples, open problems, and a wide span of algebraic and homological ideas, methods, techniques and tricks, it succeeds admirably in these aims.

After a broad first chapter summarizing background material, the book launches into a study of the basis of coherence, with ring-theoretic, module-theoretic and homological characterizations of coherent rings. The central results here are those of S. U. Chase[Trans. Amer. Math. Soc. **97** (1960), 457–473; MR0120260] and E. Matlis[Canad. J. Math. **34** (1982), no. 6, 1240–1244; MR0678666]. Chapter 3 collects together many of the topics needed to discuss more recent work, viz. change of rings, homological dimensions, Fitting invariants, Euler characteristics and Koszul complexes (for this last item the typography is cluttered and somewhat difficult to read). The next two chapters present topics in ring extensions and ring constructions such as flat epimorphisms, trivial extensions and the λ-dimension of Vasconcelos and Roos, $D+M$ constructions, and an overring approach to the study of integral closure, with attention focused on “pairs” of rings satisfying various relevant constraints. Chapter 6 looks at particular coherent rings, with a discussion of uniform coherence, of regular coherent rings, and of rings of global and weak dimension two. This last section forms an excellent preparation for those wishing to tackle the almost preternaturally slick work of W. V. Vasconcelos[*The rings of dimension two*, Dekker, New York, 1976; MR0427290].

Alfonsi’s theory on non-Noetherian grade lies at the heart of the next chapter. This allows a generalization of the Buchsbaum-Eisenbud exactness criterion, which in turn leads to Alfonsi’s reduction theorem: this enables many questions about (stable) coherence to be reduced to the case where the base ring is local. The author cleverly exploits this reduction theorem and the technique of Cartesian squares to give quick proofs of results on stable coherence and on uppers to zero in polynomial rings over coherent rings. The book closes with a discussion of coherence in the context of power series rings, group algebras and symmetric algebras.

The author of this excellent book makes a persuasive case that the study of coherent rings has fed back into the “classical” theory, suggesting new directions of research and creating many open problems. She shows that coherent rings arise naturally in, e.g., the context of polynomial rings over valuation domains, or of the study of when the integral closure of a domain is a Prüfer ring.

Of course there are cavils: there are some puzzling misprints (and one amusing one: “In 1964, Macaulay had given. . .”); Lemma 5.4.4 is needed in the proof of the preceding Theorem 5.4.3; the punctuation is rather haphazard, with the result that the meaning of sentences is not immediately clear; and results are quoted from lengthy papers or very chunky books (e.g. Bass's *Algebraic K-theory*) without specific page numbers being given. The pace too can be a little uneven; e.g. the elementary but crucial topic of latent non-zerodivisors is used without mention in Chapter 7 [cf. D. G. Northcott, *Finite free resolutions*, Cambridge Univ. Press, Cambridge, 1976; MR0460383] and the discussion of projective modules of rank one could have been a bit more detailed.

But the reader will survive these minor hazards and learn much from this book, which is a rich source of concepts, techniques, methods and ideas. *Liam O'Carroll*