Enheduanna: Princess, Priestess, Poet, and Mathematician

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Years Ago *features essays by historians and mathematicians that take us back in time. Whether addressing special topics or general trends, individual mathematicians or "schools" (as in schools of fish), the idea is always the same: to shed new light on the mathematics of the past. Submissions are welcome.*

➢ Submissions should be uploaded to http://tmin.edmgr.com or sent directly to **Jemma Lorenat**, e-mail: jlorenat@pitzer.edu.

This paper brings together a variety of poems and poem fragments written in ancient Mesopotamia that allow us to understand the origins of mathematics in its historical, social, and cultural context. Enheduanna, Mesopotamian high priestess of the moon god, Nanna, and daughter of King Sargon of Akkad, is the first recorded author in world history. The paper is organized around my translations of four of Enheduanna's temple hymns, each of which highlights a different aspect of early mathematics. Among these, *Temple Hymn 42* stands out as one of the oldest historical sources that holds a mirror to what mathematics meant for the people of that era, and it also depicts the mathematicians of the day and suggests what their job was. The poetry of ancient Mesopotamia portrays mathematics as an important activity, one worth mentioning, perhaps even bragging about, in a religious hymn—an activity that was not separate from hymn writing or from spiritual duties and observances. The paper considers the possibility that Enheduanna herself might had been a mathematician, and as such, the first mathematician to be known by name, and it concludes with my poem "Enheduanna"—a tribute to this extraordinary woman.

Brief Historical Overview

Mesopotamia (from the Greek, "between the rivers") refers to the fertile plain between the Tigris and the Euphrates Rivers, situated around the territory of present-days Iraq. The region has been the center of successive civilizations since the Neolithic period. The main civilizations after the fifth millennium BCE were in turn the Sumerian, the Akkadian, and the Babylonian.

The Sumerian civilization flourished around 3500 BCE. It was an advanced society, building cities and fortifications, and supporting agriculture, trade, and city life with a sophisticated irrigation system, canals for transportation, a legal system, administration, schools, and even a postal service. Mesopotamia is often referred to as the "cradle of civilization," the place were human beings took their first steps toward a civilized society and left a written record of their accomplishments. Writing was invented by the Sumerians around 3500 BCE. It began as pictographs, similar to Egyptian hieroglyphics, but it soon changed into cuneiform script—a phonetic variant that accommodated the natural writing material of the region—malleable clay. Cuneiform signs are wedge-shaped figures, and writing consisted in imprinting the signs with a sharpened reed or a stylus on soft clay tablets, which were then hardened by being left in the sun or baked in hot kilns.

In 2334 BCE, the Akkadian king Sargon conquered the region and ruled it until 2279 BCE. The Akkadian civilization, initially less advanced than the Sumerian, assimilated

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and continued the Sumerian religious, cultural, and scientific traditions. King Sargon started a dynasty and founded the world's first empire. Although his empire was embattled and periodically taken over by other city-state rulers, the Sumerian–Akkadian civilization lasted till around 2000 BCE, when the region came under the rule of a new nation, the Babylonians. The Babylonians adopted many of the religious and cultural customs of the Sumerian–Akkadian civilization, including the use of the Sumerian and Akkadian languages. The original Sumerian cuneiform script was adapted for the writing of the Akkadian dialects spoken throughout the region—although Sumerian remained the language of scholarship and literature—until the decline of the Babylonian empire in the second century BCE. Cuneiform writing was gradually replaced by the Phoenician alphabet around 800 BCE. By the second century CE, the cuneiform script had become extinct, and all knowledge of how to read it was lost until it began to be deciphered in the late eighteenth century [1, 9, 34].

The first major breakthrough in deciphering cuneiform script was made by the British East India Company army officer Henry Creswicke Rawlinson (1810–1895). This was followed by the deciphering of many of the cuneiform tablets unearthed by that time, but a large number of unusual tablets at the British Museum, the Louvre, Yale and Columbia Universities, and the University of Pennsylvania resisted decoding until the mid-twentieth century, when Austrian-American mathematician and historian of science Otto Eduard Neugebauer (1899–1990) discovered that these tablets contained mathematical tables and text. It is estimated today that there are at least half a million Mesopotamian clay tablets, most of them the size of a palm, scattered among museums in various countries. The ongoing deciphering efforts of a large number of scholars make available to us a considerable amount of information about these ancient civilizations' contributions to mathematics [9, 30, 34, 38, 40, 41], as well as a few poetic masterpieces offering tantalizing glimpses of mathematical activities [6, 13, 27, 28, 32, 43]. The mathematical and poetic gifts left to us by ancient civilizations seem often to be intertwined.

Among the fascinating artifacts found in the extensive excavations carried out in the region is a calcite disk from the Sumerian city of Ur (Figure 1), which introduced the world to the remarkable figure of Enheduanna (ca. 2300 BCE), high priestess of the moon god, Nanna, and daughter of King Sargon.

Religion was central to the Mesopotamians, since they believed that the gods affected every aspect of human life. As intermediaries between the gods and the people, the priests were in positions of power and responsibility. Enheduanna's appointment as the high priestess of the moon god, Nanna, patron deity of the ancient city of Ur, was a political maneuver made by her father, King Sargon of Akkad, in order to enable her to meld the Sumerian gods with the Akkadian gods to create the stability his empire required. The *En* in her name signifies her position as high priestess, *hedu* means "ornament," and *anna* means "of heaven." This was the name she assumed on her appointment as en-priestess and the only name by which we know her [25, Enheduanna]. As an en-priestess she was considered to be the earthly manifestation of Nanna's divine spouse. Enheduanna continued in this position for over forty years [28], spanning the reigns of several Akkadian kings. Upon her death, she attained semi-divine status [45]. Even in modern eyes, she is a mythical and heroic figure, whose image took hold of both the popular and scholarly imaginations and gave rise to numerous interesting works (a small sample appears in the references for this paper, and many more may be found using a search engine). Today, she is best known for her literary output, *Inninsagurra* ("Lady of Largest Heart") [6, 27, 44], *Ninmesarra* ("The Exaltation of Inanna") [6, 18, 27], *Inninmehusa* ("Inanna and Ebih") [6, 27], and the fragmentary *A balbale to Nanna* ("Hymn to Nanna") [6, 48], as well as a collection of 42 shorter pieces called collectively *The Temple Hymns* [6, 26, 28, 43]. In these works, Enheduanna distinguishes herself as a poet, not only by creating verses of beauty and power, but also by departing from the customary anonymous writing tradition of her time and identifying herself as the author of her own verses.

Figure 1. Calcite disk, Ur, circa 2300 BCE, found in a 1927 archaeological excavation that uncovered a temple complex dedicated to the moon god, Nanna. The inscription on the back of the disk identifies the central figure as Enheduanna, daughter of King Sargon, consort of Nanna. Enheduanna is dressed in the high priestess's ceremonial robes. The robed clean-shaven priest in front of her pours a libation on an altar. The cone-shaped stepped figure on the left represents the traditional temple architecture of that period, a ziggurat. University of Pennsylvania Museum of Archaeology and Anthropology, Philadelphia, Pennsylvania. (*Image courtesy of the Penn Museum, Image #295918, Object B16665.*)

(Michalowski [31] offers an interesting discussion on the customs of literary authorship in ancient Mesopotamia.) As mentioned above, Enheduanna is the first recorded author in history.

Since the time span for the events and literary and mathematical accomplishments covered in this paper is about 5000 years, for the reader's convenience, an appendix providing a timeline is included at the end of the paper. The Old Akkadian period (ca. 2350–2200), during which Enheduanna lived and composed her poetry, was a time of transition in the development of Mesopotamian mathematics. Its beginnings lay in the almost proto-mathematical stage of metrological numeration systems, and its ending heralded the adoption of the sexagesimal place-value number system and the flourishing mathematics of the Babylonian period. As a result, Enheduanna's poetry is in a unique position to mirror the driving forces behind that transition, reflecting the attitude, the motivation, and the needs that brought on the transition, as well as providing a glimpse of future developments. Enheduanna's temple hymns that touch on mathematics, as well as the other poem fragments appearing in this paper, allow us to hear the story of mathematics' complex beginnings in the voice of a culture that experienced it—as much as this is possible to do in a different language and several thousand years removed from that time.

Temple Hymn 42

Enheduanna's *Temple Hymns* have been translated into English by a number of authors [6, 26, 28, 43]. Although they were originally written in a language other than English, the word "translation" is somewhat misleading. Many of the poems were found on a number of fragments of clay tablets, each containing a truncated piece of a whole, requiring the work of a puzzle constructor to glue them together into a coherent entity. It is only then that translation became possible, starting with a transliteration and then turning the transliteration into a poem in English. No translation of a poem from one language to another can reproduce a poem exactly as originally intended, and much less so with translations from such ancient sources. Uncertainties have inevitably found their way into all the translations of Enheduanna's poetic output. The choice was sometimes between leaving a gap where a piece of the clay tablet was missing or badly damaged, choosing between the different words appearing in the numerous extant versions of a hymn, or filling in the gap with an educated guess. This paper contains my translations of four of Enheduanna's temple hymns (*Temple Hymns 1, 8, 16,* and *42*). I benefited from all the other translations but followed none exactly. In particular, I used the verbatim prose translations found in [6] and [43] (which include line numbering), and the discussions about the various meanings of Sumerian words found in all the translations, to render the poetic versions included in this paper. In my translations, I made a point of preserving the hymns' original line-counts (line-counts become apparent when one compares transliterations and verbatim translations). This mathematical structural element of the hymns has significance for at least *Temple Hymn 42*, a point that is addressed in the next section. Below is my translation of *Temple Hymn 42*. I am indebted to Sjoberg and Bergmann [43] for the last two lines of the hymn (words in brackets were inserted in translation).

Temple Hymn 42

O Ezagin, house of lapis lazuli, bright like the stars of heaven,
welcoming shrine in Eresh!

All primeval lords look up to you every month.
The great Nanigbal, Nisaba,
brought down divine powers from heaven
and added them to your own,
set up your sanctuary ready for praise.

The true woman of unsurpassed wisdom,
who cools the brows of the black-headed people,
consults a tablet of lapis lazuli, dispenses council to all lands.
True woman, pure as the soapwort, sprout of the holy reed.
She measures the heavens above
and stretches the measuring cord on the earth.
Nisaba, be praised!

The compiler of this tablet, Enheduanna.
My lord, that which has been created [here] no one has created [before].

14 lines for the house of Nisaba in Eresh

Enheduanna's *Temple Hymn 42* (Figure 2) is the last in a series of short hymns composed to various major temples of Mesopotamian deities. Betty De Shong Meador observed that "Each hymn is written to the temple itself, as though it were a living being with power and influence over its divine occupant" [28]. Most hymns mention the deity inhabiting the temple by its Sumerian name rather than its Akkadian counterpart, or on occasion a hymn uses both names and reconciles any differences by describing their attributes as a blend of the characteristics of both divinities. This performs, in effect, a unification of the Akkadian and Sumerian religious cultures, as was probably intended. For example, in *Temple Hymn 8*, dedicated to the temple of the moon god in Ur, Enheduanna's own domain (see the section "Measuring the Heavens"), she calls the moon god by both his Sumerian name, Nanna, and his Akkadian name, Suen. This became a common practice for liturgical hymns throughout the Babylonian period (see, for example, *The Herds of Nanna*, below).

Temple Hymn 42 is dedicated to Ezagin ("House of Lapis Lazuli"), the temple of the goddess Nisaba in the city of Eresh. Nisaba, also known as Nanibgal, was the Sumerian goddess of writing and mathematical calculations, patron goddess of learning and creative intellectual achievements in the arts, sciences, and literature. The impetus to the development of writing in Mesopotamia, including letters and numbers, was the need to keep records of the amassing wealth that started

Figure 2. Tablet O = N 4179 (Pl. XXXI, copied by E. Bergmann S.J.). The reverse of the tablet (shown) contains Temple Hymn 42, including the two "signature" lines at the end of the hymn where Enheduanna claims authorship. (*From "The Collection of Sumerian Temple Hymns," by Ake W. Sjoberg and E. Bergmann S.J. [43].*)

accumulating with the change from a nomadic hunting and gathering society to a settled agricultural one [25, Nisaba]. Nisaba, initially worshiped as the grain goddess, changed into the goddess of record keeping of the amount of grain in the granaries. As writing and mathematics developed and began to be used widely for both practical and spiritual purposes, Nisaba's position in the Mesopotamian pantheon became more prestigious. At the height of her powers, Nisaba, as the patron deity of scribes, was considered the scribe of the gods. Her major temple was in Eresh, a city whose location remains unknown, but she was worshipped in the temples of other gods as well [25, Nisaba]. An accomplishment in the scribal arts, be it literary or mathematical, was often accompanied by the words "Nisaba be praised!" or described as an achievement of Nisaba herself—the goddess was given her full due for divine inspiration. This is what Enheduanna has done at the end of *Temple Hymn 42* for her literary accomplishment.

Worth noting is the Sumerian hymn *The Herds of Nanna* (author unknown), written about 500 years after Enheduanna's temple hymns, ca. 1800 BCE, which provides a rare example of a Mesopotamian poem giving credit to Nisaba for a mathematical accomplishment. The excerpt from *The Herds of Nanna*, given below, is my slight adaptation of the prose translation given in [6].

> His lofty *gipar* shrines number four.
> There are four cattle pens which he has established for him.
> His great temple cattle pens, one *ece* in size, number four.
>
> The cattle are driven together in herds for him.
> His suckling calves number 39600.
> His fattened cows number 108000.
> His young bulls number 126000.
> The sparkling-eyed cows number 50400.
> The white cows number 126000.
> The cows for the evening meal are in four groups of five each.
> Such are the various type of cow of father Nanna.
>
> The herds of cattle are seven.
> Their herdsmen are seven.
> There are four of those who dwell among the cows.
> They play for him on the churn.
>
> They give praise to the lord, singing paeans as they move into the *gipar.*
> Nisaba has taken their grand total.
> Nisaba has taken their count, and she is writing it on clay.
> The holy cows of Nanna, cherished by the youth Suen, be praised!

Returning to *Temple Hymn 42*, we note that the first seven lines address Ezagin, Nisaba's temple in Eresh, in the second person; the last seven lines include a description of the role and the responsibilities of the "true woman," which is the goddess Nisaba, Enheduanna herself, or both—a point addressed in the concluding section of this paper. The description begins by presenting the true woman's wisdom and her role as interpreter of knowledge written on tablets of lapis lazuli (probably for divination purposes), and "cools the brows" may mean "heals the ill," an activity that was known to be held at temples and required much creativity (as it often does even nowadays). "The black-headed people" is the name by which the Sumerians called themselves. Next, it refers to the true woman as having successfully gone through purification rites and become as pure as the plant from which soap was made, soapwort. It also describes her as being "born," sprouting, from the reed, the holy plant out of which the writing implement was made. Lines twelve and thirteen describe the true woman's role as a mathematician. In subsequent sections we will address these two lines in more detail.

A two-line "signature," in which Enheduanna states that she is the author of the 42 temple hymns, is added at the end of the hymn. For these two lines I used the translation by Sjoberg and Bergmann [43], which appeals to me for its play on words and musicality. But it is worth noting the somewhat different translation of the signature lines by

Meador [28] for voicing Enheduanna's views on the process of poetic inspiration. Enheduanna characterizes her creative labor as giving birth, "conceiving the word." This view is expressed in another one of her works, *The Exaltation of Inanna* [5, 27]. Here is the translation of the two signature lines given in [28]:

the person who bound this tablet together
is Enheduanna
my king something never before created
did not this one give birth to it

In the next section, we begin to unravel the connections between Mesopotamian mathematics and *Temple Hymn 42* with a discussion about numbers—their development as mathematical entities, as well as the nature of their appearance in Enheduanna's temple hymns and in particular their use in the structure of *Temple Hymn 42*.

Numbers

Mesopotamian mathematics began with the development of a number system with which arithmetic became possible. This was proceeded by a long period during which numbers were intricately linked to mythology and mysticism. Perhaps a long germination period is necessary before a scientific approach to any subject becomes possible. During the mystical period, numbers became associated with the gods and were used extensively in divination. In particular, we owe some of our modern-day number superstitions to ancient civilizations, for example, the belief that 7 is a lucky number. D. E. Smith [46] mentioned that 3 and 7 have been chief among mystic numbers in all times and among all people and explained the reasons as lying "in the fact that 3 and 7 are the first prime numbers [that are] odd, unfactorable, unconnected with any common radix, possessed of various peculiar properties .... " The number 7 was important for the Sumerians. In Sumerian religion, the most powerful and important deities, the ones who decreed the fates, were the group of seven gods: Anu (sky), Enlil (air), Enki (water), Ninhursag (earth), Nanna (moon), Utu (sun), and Inanna (the planet Venus). In addition, according to ancient Sumerian mythology, 7 was also the number of the primordial beings (primeval lords) who gave birth to the gods and were usurped by their own progeny.

Several other instances of numbers associated with the gods heralded the beginning of Mesopotamian mathematics: the number 60, which was to become the base of the Mesopotamian sexagesimal number system, was the revered numeric symbol of the god Anu, the sky god, who was later viewed as the father of all gods and the ruler of the heavens [32]. Certain other important gods were given numeric symbols, among them the moon god, Nanna, whose numeric symbol was 30, perhaps because he was associated with the number of days in a lunar cycle (which is approximately 30) [25, Nanna], or as Anu's grandson, he was considered to be half of Anu.

An interesting indirect appearance of the number 60, which also includes fractions, can be found in the oldest known literary work, the Mesopotamian poetic masterpiece *The Epic of Gilgamesh* (author unknown). Gilgamesh was an actual historical figure; he appears in the Sumerian list of kings as the fifth ruler of the first dynasty of the city-state Uruk (not to be confused with Ur). He ruled Uruk around 2750 BCE, and excavations have unearthed fragments of the epic written about him soon thereafter, although the more complete versions date from later Babylonian periods. *The Epic of Gilgamesh* has been beautifully translated into English [13, 32]. In the epic, Gilgamesh is described as "two-thirds of him god and one third human" [13], by virtue of being the son of a goddess and a mortal father (who was himself the son of a goddess and a mortal father). It is difficult to figure out how this particular percentage of Gilgamesh's divinity was calculated. However, the fraction *two-thirds* appears in two other instances in the epic. In one of those instances, it occurs as a name, Urshankabi. *Urshankabi*, the name of the boatmen who ferried Gilgamesh across the Waters of Death, means "servant of two-thirds," that is, servant of the god Enki, whose numeric symbol was 40, which is two-thirds of Anu's 60 [32]. The other instance occurs in the telling of the Mesopotamian version of the story of the deluge as told to Gilgamesh by Utnapishtim (also called Utanapishti), the Mesopotamian Noah. According to Utnapishtim, the god Enki told him to build the boat with which he and those he took with him survived the deluge. The story is marvelously full of numbers and measurements, which are part of the instructions given by Enki for the construction of the boat and the timing for its completion, as well as the execution of these instructions by Utnapishtim. In the end, Utnapishtim says [13]:

At sunrise I set my hand to the oiling,
before the sunset the boat was complete.
From back to front we moved poles for the slipway,
until two-thirds of the boat had entered the water.

Thus, the fraction *two-thirds* has once again appeared in connection with the god Enki, who held an especially important position in the Mesopotamian pantheon. He was the god of fecund waters [28], the god of fresh water from underground aquifers, without which life would not have been possible in the parched land of Mesopotamia. Perhaps the two-thirds divinity attributed to Gilgamesh is intended to show that he stands in relation to the gods as Enki stands in relation to Anu, a great compliment, indeed. This brings to mind the possibility that the Mesopotamians picked the number 60 as the base of their number system because 60 has many divisors, and that allows easy representation of many fractions, which could then be assigned as numeric symbols to gods. Other speculative reasons for the choice of 60 as a base appear in [9].

At least ten different numeration systems were in use in Mesopotamia after 3500 BCE. Around 2000 BCE, the region adopted a standard numeration system that was particularly conductive to arithmetic operations and that became the driving force behind the Babylonians' mathematical achievements. This was a sexagesimal place-value number system that used only two cuneiform symbols (a symbol for 1 and a symbol for 10) to represent all numbers. In a place-value number system, the value of any single digit depends on its placement in the number. To avoid ambiguity of notation, a place-value system depends heavily on a symbol for absence —a “zero.”

Our present decimal system, including the nine symbols for the single digits and the symbol for zero, is due to the mathematicians of ancient India, where it was invented around 500 CE [3, 9, 35]. The Babylonian sexagesimal system, the earliest known place-value number system, was in use for a long time without a notation for an empty space. The value of a number whose digit placement was ambiguous was determined by context. The Babylonians seemed to have handled the ambiguity very well for about 1000 years before a number of empty space symbols began to make sporadic appearances in their numerical notation [3, 9, 35]. A recent paper by Christine Proust on mathematical education practices in Mesopotamia provides a fascinating account of how scribes were taught to carry out calculations in ways that made the inherent ambiguity useful [38]. By contrast, the numerical system developed around the same time by the ancient Egyptians was not a place-value system. Arithmetic in such number systems is much more cumbersome. As a result, ancient Egyptian mathematics developed through the advancement of geometry, which relies on concrete visual elements, while Babylonian mathematics made its progress through a form of computation that was a precursor to algebra, albeit mixed with geometric considerations, and which led to results of a more abstract nature.

Mathematical tablets found at various excavation sites in Mesopotamia include extensive tables of fractions, reciprocals, square roots, and more. At the height of the Babylonian period we encounter tablets that show the ability to solve linear equations, systems of linear equations, quadratic equations, and special cases of cubic equations, as well as to extract roots [9]. Particularly fascinating is the old Babylonian tablet *YBC 7389* (Figure 3) (ca. 1800–1600 BCE), which exhibits a very high precision approximation of the square root of 2, namely $\sqrt{2} = 1.41421356$. The method by which this number was calculated is a matter of speculation, as is the answer to the question whether the Babylonians were aware that this was only an approximation of $\sqrt{2}$ and that the decimal expansion (in this case, of course, the sexagesimal expansion) is infinite [1, 2].

Figure 3. YBC 7289, clay tablet (obverse), southern Mesopotamia, circa 1800–1600 BCE, drawing of a square with diagonal and inscribed numbers that shows a very high precision approximation of the square root of 2. YPME 007329 (BC.021354), Yale University Babylonian Collection. (*Image courtesy of the Peabody Museum of Natural History, Yale University.*)

Enheduanna’s temple hymns were written over one hundred years before the sexagesimal system was adopted as the standard for calculations, but it is hard to pinpoint the exact time at which the system came into being or when it began to gain in popularity. The numbers appearing in Enheduanna’s temple hymns reflect their mystical past as well as their mathematical future—the beginning of the sexagesimal place-value number system.

Several of Enheduanna’s temple hymns contain numbers. Predominant among them is the number 7, which appears in five out of the forty-two temple hymns. Below is my translation of *Temple Hymn 16*, dedicated to Eanna (“House of Heaven”), the temple of Inanna in Uruk, in which the number 7 occurs three times. Other translations of this hymn can be found in [6, 28, 43].

Temple Hymn 16

O house of Kulab divine powers,
perfectly shaped ripe fruit from the heart of heaven,
shrine built for the bull.

Eanna, house of seven corners,
seven fires lit at nightfall, seven flames of desire.
Your princess is on the horizon!

Your lady, Inanna,
who makes women alluring and places the war helmet on the heads of men,
the great dragon of Nigingar, queen of heaven and earth,

has erected this house on your resplendent site
and took her seat upon your dais.

11 lines for the house of Inanna in Uruk.

Inanna was the goddess of love and war, an amalgam of the gentle Sumerian goddess of love (by the same name) and the Akkadian goddess of war, Ishtar. Her astral embodiment was the planet Venus, an aspect we will take up in more detail in the next section. Inanna was the patron deity of Uruk, and her main temple, Eanna, was located in Uruk’s Kulab precinct [27]. The hymn touches on all the three aspects of the goddess: as the goddess of erotic love, she fans the “seven flames of desire”; as the war goddess, she “places the war helmet on the heads of men”; and as an astral body, she appears “on the horizon.” And in all her three aspects she is as unpredictable as a cosmic turbulence, a fire-breathing dragon, “the great dragon of Nigingar.” The number 7 makes an appearance in another of the temple hymns dedicated to Inanna, *Temple Hymn 40* (the Akkad temple of Inanna), where Inanna, as the goddess of war, is said to “prepare the seven maces” [28]. The only other known association Inanna has with the number 7 is her membership in the group of the seven gods who decree fates. The number 7 appears in three others of Enheduanna's temple hymns, only one of which is addressed to another god who belongs to the select group of 7, the god Nanna. This hymn, *Temple Hymn 8*, dedicated to Ekishnugal, the temple of Nanna in Ur, Enheduanna's own domain, is presented in the following section, where we also briefly address the numbers appearing in it.

Enheduanna's writing is carefully structured. Although the numeration in the titles of the 42 temple hymns may have been added in translations, it reflects the logical order of the original organization (motivated by religious, political, and geographical considerations) [28]. Therefore, there is every reason to believe that the structures of the hymns themselves are not arbitrary. Given Enheduanna's apparent fondness for the number 7 and her special indebtedness to the goddess of creativity, Nisaba, it seems that there is no coincidence that the number 7 plays a role in the structure of *Temple Hymn 42*: the number of lines in the hymn is $14 = 2 \times 7$, the first seven address the temple, and the latter seven describe the role and responsibilities of the "true woman." The number of the hymn himself is a multiple of 7, as is the number of hymns in the entire collection. The number 7 runs like a thread through Enheduanna's temple hymns.

Other numbers are everywhere evident as well. Each hymn ends with a line count. This shows a love of numbers almost as much as the love of words that drove the writing of the hymns. It is in connection with the count of lines in the temple hymns that we encounter an early, perhaps a first, instance of the sexagesimal number system in Mesopotamian literature. Following the last hymn in the collection appears the following inscription [43]:

> gi$_4$-ba é-n$_6$-nir
> mu-šid-bi-im ŠU+N[IGI]N-bi 8 [× 60]

Translating this transliteration, we obtain:

> Incipit: é-n$_6$-nir
> The count of lines is altogether 8 [× 60]

The word *Incipit* means that the first word of the collection of hymns, the place where the count of lines begins, is *é-n$_6$-nir* ("Eunir"). The second line totals the number of lines in the collection and finds that it is 8 (words in square brackets were inserted in transliteration and translation [43]). Using the sexagesimal system with no symbol denoting an empty space (no zero), we need to guess by the context whether the total is 8, 8×60, 8×60^2, etc. The choice is clearly $8 \times 60 = 480$ lines. Unfortunately, this total cannot be verified, since some hymns are missing lines (even the last count lines), and at least one hymn was added at a later period. But regardless, this surprisingly early encounter with the sexagesimal system is quite amazing.

Measuring the Heavens

Line twelve of Enheduanna's *Temple Hymn 42* describes one of the activities that the "true woman" was engaged in as a mathematician: "She measures the heavens above." This refers to time reckoning, an activity that was very much connected to the sky. E. C. Krupp writes:

> For most of the history of humankind, going back to stone age time, the sky has served as a tool. Just as the hands of the first people grasped the flint they crafted, so their brains grasped the sky. The regularity of the motions of celestial objects enabled them to orient themselves in time and space [21].

The sky was "the heavens above," both the home and the actual bodies of the god Anu and his divine progeny. The god Enlil, the air, through which humans saw the sky, was Anu's son. The most important celestial body for ancient time reckoning was Enlil's son, the god Nanna, the moon. Two other luminous celestial bodies also played an important role in navigating through life, time, and space—the twin children of the moon god, Nanna; Utu, the sun; and Inanna, the planet Venus—as did the entire brood of stars dotting the night skies. This was the family tree of the gods who were instrumental in the development of the Mesopotamian calendar and the beginning of astronomy during the Sumerian and Akkadian periods.

The Mesopotamians were more concerned with large-scale time reckoning than with the daily passage of hours. Nevertheless, the activities of civilized society required counting time on this small scale as well. On a daily scale, the Sumerians measured time using sundials, which was effective only during daylight hours. The day was divided into 12 periods, and each period into 30 parts. Time measurements improved with the invention of the water clock and the introduction of a more refined division of the day.

In Tablet IX of *The Epic of Gilgamesh*, we find Gilgamesh being advised to run as fast as he could to reach the end of a tunnel [32]:

> … into which the sun
> plunges when it sets and moves through the earth.
> Inside the tunnel there is total darkness:
> Deep in the darkness with no light at all.
>
> You must run through the tunnel faster than the wind.
> You have just 12 hours. If you don't emerge
> from the tunnel before the sun sets and enters,
> you will find no refuge from its deadly fire.

There follows a passage in which Gilgamesh's run is monitored and recorded hour by hour for the twelve hours of daylight before the sun enters the tunnel. In [13], the "hours" are translated as "double-hours," hinting at the duplication of the 12 daylight hours to complete the 24-hour day. This seems to indicate that the division of the day into 24 hours, 12 of which were daylight on that particular occasion, was already in place around 2000 BCE and may have been in place even earlier. It is difficult to date the Mesopotamian early time-measuring devices or the precise division of the day into smaller increments, but it is known that water clocks were in use for administrative and divination purposes during the Babylonian period, around

1300 BCE (depending on the somewhat controversial dating of the various astronomical tablets of the collection called MUL.APIN) [8]. Our division of an hour into 60 minutes and a minute into 60 seconds is a distant echo of the Mesopotamian sexagesimal number system. On a slightly larger scale, the week was set to contain seven days [19]. Once again, Anu's sacred numeric symbol, 60, and its divisors, as well as the special number 7, are employed to help with the task of developing civilization.

Another interesting point made in this passage from *The Epic of Gilgamesh* is the choice of the day: a day of the year on which daytime and nighttime are exactly equal. Such a moment, when the sun crosses the celestial equator—the equinox—occurs twice a year, in late March and in late September, and recognizing this fact requires at least careful observations and recording of the hours of daylight over extended periods of time. In other words, realizing that equinoxes exist and are recurrent phenomena requires some elementary mathematics as well as some basic astronomical observations of the sun's behavior. As we will see later in this section, equinoxes played a role in the larger-scale time measurement of the Mesopotamians, their calendrical calculations. But in this respect, the predominant astral body whose cyclic and predictable behavior enabled the Mesopotamians to develop their calendar was not the sun, but the moon.

The Mesopotamian calendar was primarily lunar. Each month began with the first sighting of the new waxing crescent moon, just after sunset on the western horizon. As a result, each month was either 29 or 30 days long, and the year was approximately 354 days. Ritual celebrations and agricultural events, though, followed the seasons, and thus were solar in nature. The solar year's length is just over 365 days, and intercalary months were added when necessary to reconcile the lunar and solar years. During the Babylonian period, the lunar calendar required an intercalary month three times every eight years along with other periodic adjustments, as ordered by the king, to keep up with the 365-day solar year. It took till about 380 BCE to standardize the intercalation practices. Before that, calendars, names of months, and dates of festivals differed from city to city. The equinoxes played a significant role in all the Mesopotamian calendars. The year seemed to be the union of two six-month "equinox years," and the new year was celebrated twice annually, on each occurrence of the vernal or autumnal equinox [20].

It was the task of the priesthood in each city to recommend calendrical adjustments and improvements, to observe and record appearances of the new moon, and to calculate the times of the equinoxes, as well as to predict celestial events such as eclipses and comets, all of which were viewed as signs from the gods. The latter seemed to have been an all-important and influential activity in Mesopotamia, since it was the only way to read the wants and needs of the gods directly from the sky. The practice of astronomy, along with the closely related practice of astrology, was secret and conducted by the temple mathematicians, who were members of the priesthood trained in the arts of mathematics and divination [20, 41].

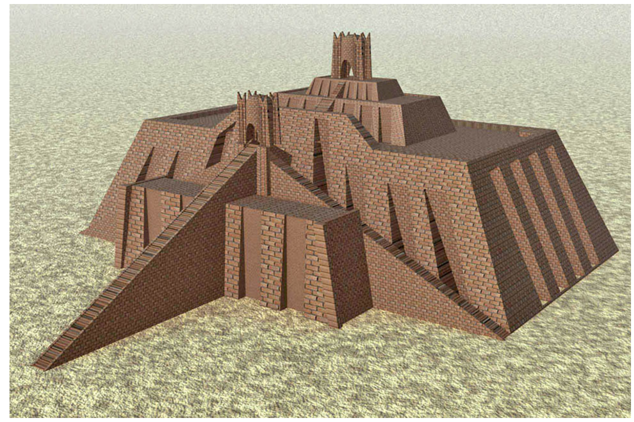

Figure 4. The Great Ziggurat of Ur. Reconstruction of Ur-Nammu's ziggurat based on the 1939 reconstruction drawn by Sir Leonard C. Wolley. This ziggurat, built around 2100 BCE, was part of the temple complex for the worship of the moon god, Nanna. It was unearthed in the 1920s in the excavations of Ur led by Sir Leonard C. Wolley. (*Available under the Creative Commons Attribution-ShareAlike License at* https://en.wikipedia.org/wiki/Ziggurat_of_Ur.)

They engaged in extensive celestial observations, calculations, and record keeping in the elevated privacy of the ziggurats [12, 20, 21]. A ziggurat (Figure 4), which was a part of every temple complex, consisted of a massive stepped pyramid, on whose top rested the holy of holies, the chamber of the god or the goddess who was worshipped at that temple.

The ancient city of Ur, established around 4000 BCE, was the seat of the major temple to Nanna, Ekishnugal ("House of Great Light"), where Enheduanna presided as high priestess. Archaeological digs in Ur have yielded tablets with the beginning of calendrical calculations from before 2334 BCE. They also unearthed more complete calendrical tablets showing names for 12 months and a 13th intercalary month from the time of King Shulgi (about 2050 BCE) [20]. The first month of each year began at the vernal equinox, the seventh month at the autumnal equinox. An interesting aspect of the Ur calendar was that the New Year festival, the *akiti*, at the autumnal equinox was a more important celebration than the New Year festival at the vernal equinox. Cohen explains the astronomical and mythological reasons behind this:

> The equinoxes began a period of disharmony between the moon and the sun. During the "equinox year" between the seventh month and the first month the moon was visible longer in the skies, the reverse during the other "equinox year." In Ur, the city of the moon, the *akiti* was a celebration of the triumph of Nanna, the moon—particularly the *akiti* of the seventh month, when Nanna would begin having visible superiority over the sun, Utu. This may be the reason the *akiti* of the seventh month at Ur appears to have been more important than that of the first month [12, 20].

Echoes of Ur's calendrical activities are found in Enheduanna's *Temple Hymn 8*. Below is my translation. Other translations of this hymn may be found in [6, 28, 43].

Temple Hymn 8

O Ur, strong bulls standing in the reed brake,
birds trapped in their nests, food for all lands.

Ekishnugal, calf of a great cow, holy light of heaven,
your shrine is the pure place where the sky touches the earth.

O house of Suen, your front looks princely, your back is fit for a king,
your banquet halls resound with *adab* hymns, *shem* and *ala* drums,
your beaming light and your steadfast lordship are precious.

In the *gipar*, the priestess quarters shining like the sun,
your princely shrine of pure divine powers marks the passage of days.

Ekishnugal, great serpent gleaming on its marsh bed,
your moonlight bright as daylight sweeps the land.
Your foundations hold fifty *abzus*,
reach to the seven seas below,
look into the hearts of the gods.

Your lord, the lord who decrees fates, the crown of heaven, Ashimbabbar,

has erected this house on your resplendent site
and took his seat upon your dais.

17 lines for the house of Nanna in Ur.

Temple Hymn 8 shows a great familiarity with the temple complex and its activities, as is to be expected from an author who is its high priestess. The hymn begins by addressing the city of Ur itself. Ur is depicted as a city of plenty, a city that generously offers food to all. The hymn then proceeds to address Ekishnugal, the temple complex, praising it and describing the superior state of its various parts and the activities taking place in each of those parts. Lines 5–7 depict the temple's banquet halls, where a feast accompanied by singing and music takes place. In lines 8–9, the hymn turns its attention to the *gipar*. The *gipar* was a cluster of several buildings in which the high priestess had her quarters. It is described as "shining like the sun," which probably means purified and adorned for celebration. It also mentions that in the *gipar*, the "princely shrine of pure divine powers marks the passage of days," which suggests that some astronomical observations or calculations, probably involving the behavior of the moon and the sun, were carried out in the priestess's quarters, either by Enheduanna herself or under her supervision. These descriptions seem to indicate that a great celebration took place in the temple complex, a celebration that involved the banquet halls, the *gipar*, and the entire city of Ur. This may indicate that celebrations exalting the moon god, Nanna, were often held in the temple complex and the city, but it may also point to a specific holiday, the autumnal equinox New Year festival, the *akiti*, when the moon begins to have visible superiority over the sun, which people experience as "moonlight bright as daylight sweeps the land." In either case, the hymn points out that it was Enheduanna's responsibility to keep track of the days for those holidays through astronomical observations and calculations.

In line 10 of the hymn, Enheduanna returns to describing and praising the temple complex she admires and knows intimately. Line 12 states that the temple's foundations hold "fifty *abzus*." *Abzu* was the Sumerian name for the domain of the god Enki, the fertile sweet water under the earth, the groundwater, the kind of water coming to the surface from the foundations of the temple that can be collected into pools. This word was also used for the purification pools filled with clean sweet water on the temple's grounds [4]. Priests were required to follow strict and extensive purification rites before approaching the gods. The number fifty seems to be unconnected to any sacred numerical symbol, and no other reference to the god Enki appears in the hymn; therefore, "fifty *abzus*" most likely refers to an actual count of purification pools in the temple complex. The word "sea" in line 13 is a translation of the Mesopotamian word *engur*. This word is sometimes used as synonymous with *abzu*, but it may also denote the primeval waters, the ancient vast sea under the earth out of which the world was created [28]. Given its connection with the sacred number 7, we assume that the latter was the intended meaning. The moon disappears from the sky for one night at the end of every month, before the new crescent moon signifying the beginning of a new month appears. Identifying the temple and its god, Enheduanna sees both as reaching great depth, "the seven seas below," at the end of a monthly journey that includes a sojourn through the greatest heights. Therefore, on completing the monthly journey, both temple and god had looked, high and low, into the very core of the mysteries—the hearts of the gods. In the next line the moon god, called by his third name, Ashimbabbar, is described as he "who decrees fates" from his lofty position as the "crown of heaven."

Astronomical observations carried out in ancient Mesopotamia were geared mainly toward reading the will of the gods. Nevertheless, they formed a solid foundation on which both the calendar and mathematical astronomy, which began to be practiced in the region toward the end of the Babylonian period, were developed. The moon received most of the attention, but many other celestial objects that could be viewed with the unaided eye were carefully observed and their behavior recorded. The tablets, named collectively "astrolabes," provide astronomical observations dated as early as 1100 BCE and believed to be of much older origin [20]. These include data on the heliacal rising of certain stars, and they exhibit the earliest division of the star-studded sky in a circular pattern—what would later result in the construction of the zodiac and eventually in the division of the circle into 360 degrees.

Another ancient collection of tablets, called "Enuma Anu Enlil," written in the seventh century BCE but believed to be of much more ancient origin, perhaps as old as the second millennium BCE, provides an extensive catalogue of omens, but also many interesting and accurate astronomical observations. A special tablet from this collection, the "Venus Tablet of Ammizaduga," records the rising and setting data for the planet Venus (Inanna) over a period of 21 years [20]. From these and other sources we know that the Mesopotamians were aware of the difference between the stars that appear in the unchanging patterns of constellations and the "wandering stars" (in modern terminology "planets," from the Greek *planētēs*, "wanderers") whose paths wander among the constellations in cyclical sequences of appearances and disappearances. In particular, they recognized Venus as a "wandering star," a celestial object of intense luminosity that appears as both the morning star and the evening star [20, 21]. It is unlikely that the detailed cycle of Venus was known during Enheduanna's time, but in *Temple Hymn 16* (see the section "Numbers"), we see a recognition that Venus behaves differently from most of the other stars. Venus is described as the "great dragon of Nigingar," whose seemingly unpredictable appearances and disappearances are noted in a hymn—a first step toward more serious astronomical observations and recording. The ancient sources provided an increasingly more scientific and accurate picture of the astral bodies' movements in relation to each other. And around the beginning of the fifth century BCE there appeared the first great Babylonian mathematical astronomer whose name and calculations have survived, Nabu-rimanni, son of Balatsu, descendant of and himself a priest of the moon god, Nanna [36].

Stretching a Measuring Cord on the Earth

Line thirteen of Enheduanna's *Temple Hymn 42* describes the second activity that the "true woman" was engaged in as a mathematician: she "stretches the measuring cord on the earth." In contrast to the sky watching required for time reckoning, this activity refers to mathematical measurements done here on earth. These included, but were not restricted to, setting of property boundaries; design of ramparts and defense fortifications, and of temples, castles, and other buildings erected in the thriving Mesopotamian metropolises; construction of irrigation and transportation canals and dikes; and various other engineering projects. Such projects involved both plane and solid geometry, for measuring areas and volumes, as well as the ability to lay down and measure a straight line. The latter was done by stretching a cord horizontally or hanging a cord vertically, weighted by a load—a plumb line. Therefore, those who were engaged in such measurements were said to "stretch the measuring cord on the earth."

The geometry of elementary shapes appears in the literature as early as *The Epic of Gilgamesh*. Below is a small excerpt from the story of the deluge (for more details, see the section "Numbers"). The first stanza, spoken by the god Enki, begins the instructions on how to construct the boat with which Utnapishtim and those he plans to take with him will survive the deluge. In the second stanza, Utnapishtim tells Gilgamesh how he followed Enki's instructions [13]:

> "The boat you will build,
> her dimensions all shall be equal:
> her length and breadth shall be the same,
> cover her with a roof, like the Ocean Below."
> ....
> By the fifth day I had set her hull in position,
> One acre was her area, ten rods the height of her sides,
> At ten rods also, the sides of her roof were each the same length.
> I set in place her body, I drew up her design.

We note that Enki's instructions to Utnapishtim were too general for the construction of the boat itself. The god's words provided the impetus, the permission, and the blessing for the actual execution, but it was people who designed the boat and built it.

Another instance of the same phenomenon occurs in Tablet I of *The Epic of Gilgamesh*, where we find verses praising the king of Uruk, Gilgamesh, for building the city's ramparts and the temple Eanna, where the goddess Inanna (called in the epic by her Akkadian name, Ishtar) was worshipped [13]:

> He built the ramparts of Uruk-the-Sheepfold,
> of holy Eanna, the sacred storehouse.
>
> See its wall like a strand of wool,
> view its parapet that none could copy!
> Take the stairway of a bygone era,
> draw near to Eanna, seat of Ishtar the goddess,
> that no later king could ever copy!

Once again, a man, Gilgamesh, albeit two-thirds divine, built the unsurpassed fortifications of Uruk and its magnificent temple to Inanna. The epic seems to imply that the gods themselves do not provide precise blueprints for these activities. Down-to-earth mathematics, although divinely motivated, is the province of humans. And indeed, this attitude is reflected in the training and practices of those engaged in mathematical activities. As mentioned in the previous section, calendrical and astronomical calculations were closely and directly connected to the gods and divination and were therefore performed by members of the priesthood. This was not the case for the type of mathematical activities that required "stretching the measuring cord on the earth." Those who "stretched the measuring cord" were individuals educated and trained for the appropriate mathematical work, without necessarily being part of the priesthood. These included professional scribes, who received a broad-based education in scribal schools, as well as individuals who were privately tutored in all or some of the scribal school subjects.

References to scribes in Mesopotamian sources seem to imply that the majority of them were men. But women, although fewer than men, were not excluded from the profession [23, 45]. The oldest reference to a woman scribe, named Nin-un-il, appears in a source dated from the Old Akkadian period [23]. The mathematics education of scribes in Mesopotamia has been explored in several recent publications, such as [38], which provides a detailed discussion of the elementary mathematics education in scribal schools, while [41, Chapter 9] provides a broad overview of the general mathematical training of scribes. In addition, [39] contains a number of papers discussing the various levels of cuneiform literacy and numeracy of both men and women in Mesopotamia.

At the height of its accomplishments, Babylonian geometry included a fairly accurate computation of the constant π, the ratio of the circumference of a circle to its diameter, specifically, $\pi = 3\frac{1}{8}$. Pictorial evidence suggests that the method employed to arrive at this result can be considered a precursor of the method used by Archimedes (ca. 250 BCE) to obtain his more accurate approximations of π. Tablets found in Susa in 1936 provide several other geometric results, including the formula for the volume of a truncated pyramid, as well as results about the geometry of triangles and circles that were believed to have been discovered by the Greeks [1, 9]. None of these results appear with proofs. The requirement that mathematical results be validated by logical proofs within an axiomatic framework, which is the cornerstone of what we think of nowadays as mathematics, was introduced by the Greeks, ca. 600 BCE [9, 35]. This does not necessarily mean that the Babylonians did not seek a form of logical justification for their mathematical results. In [11] we find several papers discussing various forms of logical justification for mathematical results obtained in antiquity. Of particular interest is Proust's paper [37], which discusses a numerical Babylonian tablet that provides an algorithm for obtaining reciprocals of a certain geometric progression of numbers in sexagesimal notation. The paper makes a convincing argument that the purpose of the tablet is not the computation, but "the algorithm itself, its operation and its justification," including the fact that "this operation is its own reverse operation."

The most famous Babylonian mathematical tablet, *Plimpton 322,* dated between 1820 BCE and 1762 BCE (Figure 5), contains sophisticated mathematics bridging between geometry and algebra and touching on results that were believed to have been discovered during the Hellenistic period, hundreds of years later. There is evidence that many ancient civilizations were aware of the result we know as the Pythagorean theorem, $c^2 = a^2 + b^2$, where c is the hypotenuse of a right triangle, and a and b are the two legs [9]. Because this theorem is attributed to Pythagoras, any three positive integers *a, b*, and *c* that satisfy this equation are called a Pythagorean triple. A formula for generating such triples was discovered during Hellenistic times by the most original mathematician of late antiquity, Diophantus of Alexandria (ca. 250 CE) [9]. In 1945, Neugebauer and Sachs [33] decoded the Babylonian clay tablet *Plimpton 322*. The text involves a table with four columns of numbers, which after careful scrutiny turned out to yield Pythagorean triples. There are 15 of them, of which the largest is 12709, 13500,

Figure 5. Plimpton 322, clay, mathematical cuneiform tablet with four columns and fifteen rows of numbers. Larsa, Mesopotamia (modern Tell Senkereh, Iraq), circa 1820–1762 BCE. (*Plimpton Collection, Rare Book & Manuscript Library, Columbia University in the City of New York.*)

18584 (the hypotenuse, *c*, and one of the legs, *b*, appear explicitly in *Plimpton 322*; the other leg, *a*, can be inferred from the rest of the data). Although no method for finding the triples appears in the tablet, it is clear that they were not discovered by chance. And a fourth column providing the ratios $(c/a)^2$ leads to the most likely conclusion that the Babylonians were aware of the formula that Diophantus rediscovered several hundred years later [9]. *Plimpton 322* was at the center of a scholarly controversy as recently as 2017 [22, 24]. Although [24] makes some exaggerated claims about the mathematics included in *Plimpton 322*, the controversy is interesting to read. It also provides a speculative, but reasonable, explanation of the practical uses of the information found in the tablet. Specifically, such Pythagorean triples provided necessary measurements for some constructions involving right triangles, such as, for example, the construction of a ziggurat. To carry out the speculation a bit further, we can imagine that constructions requiring right triangles could be carried out approximately by the builders and then had to be checked for accuracy using such a tablet of triples. The mathematician working for this building project may have stretched the cord and measured all sides of the triangle, did some minor calculations, and checked the tablet. If the measurements complied with the data in the tablet, then all was well, and the triangle was indeed right-angled. But such a conclusion is possible only if we assume that the Babylonians knew that the converse of the Pythagorean theorem holds; that is, if the equation $c^2 = a^2 + b^2$ is satisfied for a triangle with sides *a, b*, and *c*, then the triangle is right-angled and *c* is its hypotenuse. The converse of the Pythagorean theorem was first proved in Euclid's *Elements* (ca. 300 BCE) [9], but it is not out of the question that it was known to the Babylonians, without proof, hundreds of years before.

Most of the advanced geometric results described above were discovered long after Enheduanna's death. In Enheduanna's temple hymns we are offered a glimpse of the constructions that spurred these discoveries. In particular, *Temple Hymn 1* offers a poetic view of the start of temple and canal construction—two of the most important types of construction whose computational requirements contributed to the development of Mesopotamian mathematics. Below is my translation of *Temple Hymn 1*. Other translations of this hymn may be found in [6, 28, 43].

Temple Hymn 1

Eunir, first ziggurat uniting heaven and earth,
foundations of heaven and earth,
you stand high on the most holy site, Eridu.

Abzu, pure water shrine erected by your prince,
mountain, scoured clean with soapwort, where water is hauled
from your lord's pure canals and pure food is eaten.
Your sacred songs praise his divine powers!

Your walls are kept in good repair,
brick to brick firmly joined together,
even the light cannot filter into the chamber
where your revered lord dwells.
He placed a crown on your head. House without equal!

Your *susbu* priests make pure plants grow,
they call upon Utu to ripen the crops in abundance,
bake savory bread in your ovens,
prepare a feast to rival the grandest banquet halls.

O Engur, vast subterranean sea,
your prince, the prince of heaven and earth,
whose word cannot be altered,
Lord Nudimmud, wise beyond measure,
creator, life giver,

has erected this house on your resplendent site
and took his seat upon your dais.

23 lines for the house of Enki in Eridu

The hymn is dedicated to Engur (also called E-engura), the temple of the god Enki in Eridu. Enki, also named Nudimmud, whom we encountered previously as the god of fresh water from underground aquifers and the only deity who saved humanity from the deluge, was also the god of wisdom, intelligence, trickery and mischief, magic, exorcism, healing, creation, virility, fertility, arts, and crafts. This long list of spheres of influence highlights the importance of Enki and what he stood for—life-giving water. Enki, one of the deities belonging to the select group of seven gods who decreed fates, was a son of Anu (sky) and the husband of Ninhursag (earth), with whom he fathered eleven children, who were put in charge of various important aspects of civilized society [25, Enki]. His role varied through historical periods, but he was invariably portrayed as a creator of life and as a benevolent teacher of the arts of survival and civilization. Enki was the patron god of Eridu, the most ancient city in Mesopotamia. Located a few miles southwest of the Euphrates, in the marshy region where the Tigris and Euphrates merge before they flow into the Persian Gulf, Eridu was established around 5400 BCE by the first settlers in the area, the Ubaids, a nation about which we know very little, who may have been the ancestors of the Sumerians [28]. The first temple in Mesopotamia, a temple to Enki, was erected by the Ubaids in Eridu. Excavations at the site of a mount that is a remnant of a ziggurat that was part of a temple to Enki (built ca. 2100 BCE) unearthed eighteen levels of successive temple constructions. At the lowest level was the very first temple built on that site, a one-room structure no larger than three meters square, dated about 4900 BCE, containing a pedestal and a recessed niche [28]. Old Sumerian myths credit Enki with the building of the first temple in his own honor and the establishment of the city of Eridu itself [28, 42]. *Enki's Journey to Nibru* [6] depicts the enterprise:

> The lord of the *abzu*, King Enki, Enki the lord who determines fates, built up his temple entirely from silver and lapis lazuli … In Eridu, he built the house on the banks … E-engura, high citadel standing firm on the earth! Temple at the edge of the *engur*.

Citing Sumerian sources, Satterfield [42] points out that later Mesopotamian temples were founded in conformity with the "ordinances and rituals" of Eridu, although no extant text gives any details on what those ordinances and rituals were.

From the point of view of architecture and construction, that is, from the point of view of activities that require mathematical skills, Enheduanna's temple hymns, particularly *Temple Hymn 1*, reflect a number of common features of Sumerian temples that evolved from construction practices originating in deep antiquity and therefore are likely to have been a part of the ordinances of Eridu.

The first stanza of *Temple Hymn 1* mentions the ziggurat (*Eunir* means "House of the Ziggurat"), the most distinctive feature of Sumerian temples. The ziggurat "unites the heavens and earth" quite literally, starting from a deep foundation that reaches down to the aquifer, it rises upward as a stepped pyramid to the last platform, high above, where the "chamber where your revered lord dwells" sits like a "crown on your head." The third stanza of *Temple Hymn 1* mentions the construction of the walls of the ziggurat that were built with kiln-baked clay bricks and sunbaked mud bricks, which were mortared with bitumen, a naturally occurring tar [14].

The bricks were manufactured with mathematical precision. For example, each baked brick used in the construction of the Ur-Nammu ziggurat (Figure 4) measured about 11.5 × 11.5 × 2.75 inches and weighed about 33 pounds [14]. The bricks were laid with equal precision in order to provide a solid and durable wall, so that "even the light cannot filter" into the building, especially the top chamber, where the god's statue resided in splendid isolation [14]. Ziggurats increased in height, number of platforms, and elaboration of design hand in hand with the development of the mathematical and engineering skills required for their construction [14, 28]. As seen in Figure 4, a number of stairways ascended to the top. No stairways are mentioned in *Temple Hymn 1*, but the image of priests ascending toward the "revered lord's chamber" laden with purifying water and offerings of food is conjured by the description given in the second stanza, where water is hauled from the "pure canal" at ground level up the "mountain."

Lifting the water uphill may not have been accomplished entirely by manual labor. The oldest efficient water-lifting device is the *shaduf*, which consisted of an almost horizontal pole mounted like a seesaw. A bag was suspended from a rope at one end of the pole, and a counterweight was hung on the other end. The person operating the *shaduf* pulled down on the rope to lower the bag into the water, then allowed the counterweight to raise the full bag, and finally dropped the water from the bag at the higher level. To raise water through several levels, a series of *shadufs* was deployed, one mounted above the other [47]. This technique was known in Mesopotamia during Enheduanna's lifetime (as early as 2300 BCE [47]) and might be the "water hauling" scene depicted in the second stanza of *Temple Hymn 1*. Methods for hauling water uphill from a ground-level source had also improved with the development of mathematical and engineering skills. By the time of the Babylonian king Nebuchadnezzar II (604–562 BCE), the technology involved in such an architectural feature was sufficiently advanced to allow him to construct one of the so-called seven wonders of the ancient world, the Hanging Gardens of Babylon, which had the form of a terraced pyramid planted with exotic shrubs and flowers, whose "water machines," a system of pumps and pulleys, were hidden from view by foliage. According to legend, he built it to please his foreign concubine, who had a passion for mountain surroundings [10, Hanging Gardens of Babylon].

The gods' temples in Mesopotamia were elaborate multiacre complexes of buildings and land, which acted as houses of healing, divination, and counseling, food storage and distribution centers, schools, residences for priests, astronomical observatories, city administrative centers, and holy sites. There were no temple services that would be recognized as such nowadays. Individuals interacted with the gods during festivals through the mediation of priests, or at home through private rituals [28], [25, Enki]. The fourth stanza of *Temple Hymn 1* mentions the *subsu*-priests (a type of temple priest) farming the temple lands and preparing food for feasts and sacred offerings. Although, not mentioned in the hymn, such activities must have required storage buildings for crops and housing for livestock. Other poetic sources make this requirement more explicit. For example, the hymn *The Herds of Nanna,* which appears in the section "Temple Hymn 42," points out that the cattle belonging to the god were penned in the *gipar*, the quarters of the high priest or priestess at the temple. The word *gipar* in Sumerian means "storehouse," which may mean that the use of the temple's *gipar* for storage had ancient origins and most likely conformed with the ordinances of Eridu.

The "pure canals" mentioned in the second stanza of *Temple Hymn 1* offer a poetic glimpse of another type of construction requiring mathematical and engineering skills. A temple's grounds contained a number of purification *abzus*, pools of sweet water, for ritual cleansing (as mentioned in the section "Measuring the Heavens" and also discussed in the previous paragraph), as well as canals crossing the land used for agriculture. Both types of sweet-water contrivances could be called "pure canals." In Mesopotamia, especially in the southern part of the region where the Sumerians settled, agriculture was not possible without some form of man-made irrigation. Such irrigation was employed by the Ubaids as early as 5000 BCE, although it is not known what mathematical skills were involved at that stage [25]. An extensive agricultural project, such as the one that seems to be described in the fourth stanza of *Temple Hymn 1*, would have required the construction of gravity-fed irrigation canals leading from the Euphrates. Such a canal had a gentler slope than the river and was situated, after a distance, above the land being irrigated [47]. Because of the special conditions in the region, those were the type of canals used for large-scale agriculture throughout Mesopotamia since about 2900 BCE (or even earlier) [47]. The canals were also used for travel by water in times of peace, and transportation of troops in times of war. Such canals required substantial maintenance involving cooperation between two or more city-states. In *Enki and the World Order* [6] we find the god Enki sailing through the waterways from temple to temple, assigning duties to various deities. Enki divides the responsibilities for canal construction and maintenance between two deities, the god of rivers and canals, Enbilulu, "him who with glorious mouth submits to verification the devouring force of the Tigris and Euphrates, while prosperity pours forth from the palace like oil—Enbilulu, the inspector of waterways," and the god of farming, Enkimdu, "the lord who wears the diadem, the farmer of Enlil—Enkimdu, responsible for ditches and dikes." He also reminds Ishkur, the storm god, of his heavenly supervisory responsibility: "Enki placed in charge of all this him, the son of Anu, the canal inspector of heaven and earth—Ishkur, the bringer of plenty." This three-god patronage attests both to the importance of the canals in ancient Mesopotamia and to the variety of skills necessary for canal construction and maintenance.

Could Enheduanna have been involved in "stretching the measuring cord on the earth," perhaps in a supervisory role, if not quite in person? After all, the temple mathematicians had to ensure that temple constructions and repairs followed the "ordinances and rituals" of Eridu, and they may also have been involved in the geometric layout and measurement of important engineering projects such as canal building and repair. The high priestess of the most mathematical of gods, the moon god, Nanna, must have had some form of involvement in these tasks.

The True Woman

We now return to the enigmatic "true woman" that appears in *Temple Hymn 42*. There is little doubt that lines twelve and thirteen describe the mathematical responsibilities of the goddess Nisaba. The lines echo Enki's assignment of duties to Nisaba found in the myth *Enki and the World Order* [6]: "Holy Nisaba, is to get the measuring reed."

Did Enheduanna also speak about herself? We have seen in the previous sections that as high priestess of the moon god, Nanna, Enheduanna was likely to have been involved in "measuring the heavens"— the part of mathematics used for divination and calendrical calculations. We also speculated that Enheduanna might have been involved in "stretching the measuring cord on the earth." In no place in the temple hymns or in her other poetic works does she make an explicit claim to being involved in mathematical work, as she does for her poetic work. But the poetic process is mysterious, and a poem may reveal certain things about the poet not explicitly stated in it. *Temple Hymn 42* is different in a number of ways from the other hymns in the collection. To start with, it omits the two-line colophon that appears in all the other hymns:

> has erected this house on your resplendent site
> and took his/her seat upon your dais.

The colophon is replaced by the words "Nisaba be praised!" and it is followed by the two "signature lines" in which Enheduanna claims authorship of her poetic work. As such, *Temple Hymn 42* is not just a hymn praising a goddess and her temple like the others hymns in this collection; it is also a personal poem, a hymn giving thanks to the goddess of creativity for the gifts of talent and inspiration. And the thanks for talent and inspiration might have been extended for all the creative activities mentioned in the poem, and in particular for poetry and mathematics. Enheduanna's wording regarding the "true woman" is ambiguous. She does not make it clear that by "true woman" she means the goddess Nisaba. This ambiguity, either intentional or subconsciously introduced in the hymn, points to the possibility that the last seven lines may describe the responsibilities of both goddess and priestess. But of course, we will never truly know. A distance of thousands of years, cultural differences, and unavoidable translation inaccuracies make it difficult to be certain of any conclusion. I will end this paper with my poem *Enheduanna*, a tribute to this "true woman," princess, priestess, poet, and, I would like to believe, mathematician.

Enheduanna

… in the beginning there is no beginning,
eternity's dark fingers hold a lantern casting a glow
over the city-state of Ur,

where the Sumerian princess, Enheduanna, high
priestess
of the Moon God, Nanna, daughter of King Sargon,
stretches the cord measuring land and irrigation
canals, tall ziggurats
and massive ramparts.

Her other task is time reckoning. Nanna
dictates the calendar, expecting priests to iron out the
details. She measures
the high heavens to keep his lunar months in sync
with the four seasons,

and to exalt the pantheon of gods and let her heart be
appeased,
conceives the words.

Poet and mathematician—the first to sign her own
work:

The compiler of this tablet, Enheduanna.
My lord, that which has been created [here] no one
has created [before].

14 lines tribute to Enheduanna

For interested readers, more poetry inspired by mathematics and its history appears in [15–17].

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Appendix: Mesopotamia Timeline

The timeline below shows the major events and the literary and mathematical accomplishments of the Mesopotamian civilization discussed in this paper (whose time frame is approximately 5400–400 BCE). In addition to the sources cited in the text for each accomplishment or event, I have used the historical timelines from [7], [25, Mesopotamia Timeline], and [29]. The last dateable cuneiform tablet is believed to have been written about 500 years after the end of this stretch of time, ca. 70 CE.

Periods (all dates are BCE)	Selected events, works, and people
ca. 7600–3500	First shrine to the god Enki in Eridu (ca. 5400).
	Irrigation and agriculture begin in earnest (ca. 5000).
	Building of the first walled city, Uruk (ca. 4500).
Late Uruk	First city-states.
(ca. 4100–2900)	Building of Ur (ca. 4000).
	Invention of writing by the Sumerians (proto cuneiform, ca. 3500).
	The Flood (ca. 2900).
Early Dynastic	Gilgamesh, king of Uruk (ca. 2750).
(ca. 2900–2350)	Cuneiform writing fully developed.
	Earliest literary text: tablets from *The Epic of Gilgamesh* (ca. 2100).
	Metrological numeration systems.
	Earliest tablets of mathematical problem sets.

Appendix . (continued)

Periods (all dates are BCE)	Selected events, works, and people
Old Akkadian (ca. 2350–2200)	Sargon of Akkad (2323–2278) founded the first empire and dynasty.
	Enheduanna (ca. 2300), the world's first author known by name.
	Dating of the *Enheduanna Disk, B16665* (ca. 2300).
	Composition of *The Temple Hymns of Enheduanna* and her other poems.
Ur III or Sumerian Renaissance (ca. 2100–2000)	The sexagesimal place-value number system becomes the standard numeration system (ca. 2000).
	The construction of the Ur-Nammu Ziggurat (ca. 2100).
Old Babylonian (ca. 2000–1600)	Composition of *The Herds of Nanna* (ca. 1800).
	Dating of the majority of mathematical tablets.
	YBC 7289 (ca. 1600–1800).
	Plimpton 322 (ca. 1820–1762).
Kassite (ca. 1600–1100)	Standard version of *The Epic of Gilgamesh,* compiled by the Uruk scribe Sin-liqe-unninni (ca.1300–1000).
	Water clocks used for administrative purposes (ca. 1300).
Middle and Neo-Assyrian (ca. 1400–610)	Astronomical tablets: *Astrolabes* (ca. 1100 or older), MUL.APIN (ca. 687 or older).
Neo-Babylonian (ca. 625–539)	Astronomical tablets: *Enuma Anu Enlil* (seventh century; some tablets in the collection are believed to be much older), the *Venus Tablet of Ammizaduga* (ca. 612 or older).
	The construction of the Hanging Gardens of Babylon (ca. 580).
Persian and Macedonian (ca. 539–300)	Standardization of calendrical intercalation practices (ca. 383–367).
	Nabu-rimanni (fifth century), first mathematical astronomer whose name is known.

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