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What Can We Do If We Crave Certainty in Mathematics?

Thus, to salvage traditional mathematics, Hilbert proposed a bold new program. It required first that the whole of existing mathematics should be axiomatized, and second that this axiomatic theory should then be proven consistent.

— David M. Burton, *The History of Mathematics*

Hilbert said, "No one shall expel us from
the paradise that Cantor has created." And we believed,
like Cantor himself, that God had opened the gates
to the forbidden garden,
had invited us to enter,
meet א (*aleph*) face to face,
converse in the language of sets,
admire the ascent of transfinite cardinals
into an infinitude of infinities.
No one can possess such knowledge and remain unscathed.
We lost our footing; doubts assaulted us
from the very first step,
set roadblocks in our path—
unanswerable questions, inexplicable paradoxes
and baffling results.

Before Kurt Gödel, we could still have hoped.
Attempting to resolve inconsistencies,
we could have spent our lives trying to grasp the tantalizing
cloud of certainty hanging above our heads—
just beyond reach.

The naïveté of Frege, Russell, Hilbert,
and all of us, their followers,
divided into schools with bombastic titles—
Axiomatic, Logistic, Formalistic—
with methods and approaches,

plans for the future,
a list of problems to last to the end of time.
Little did we know of logic's limitations:
that our system would backfire,
stating its own incompleteness,
in its own ink signing the QED—
that with cymbals and umlauts
it would prove its inability to prove
consistency of axioms within the system.
And what else was there but the system
we took for granted
as we did our ability to breathe?

After Gödel proved the Incompleteness Theorems
a grey cloud descended upon us—
we could touch the fog.
Nothing was pure logic.
Pure logic was nothing.
We could not even count on knowing what truth
can be proved.
Uncertainty permeated everything.

We prayed that this was not the end of the road—
that there was more of it to travel.

From *Ode to Numbers*, Antrim House, 2017

Note: Georg Cantor (1845-1918) is responsible for the rigorous mathematical representation of infinity and the development of set theory – the "language" that allowed mathematicians to work with the concept of infinity. Almost immediately after its discovery, a number of paradoxes were found in set theory, among them the famous *Barber Paradox*, posed by Bertrand Russell (1872-1970). Considerable efforts were made to fix these paradoxes, including the program proposed by the prominent German mathematician, David Hilbert (1862-1943). These efforts were dealt a severe blow by the Incompleteness Theorem proved by Kurt Gödel (1906-1978). Fortunately, it was not the end of the road. Realizing the exact extent of uncertainty inherent in the mathematical system had the beneficial effect of removing vague anxieties and redirecting research focus.