Precalculus Learning Goals - Week 12

This week we’ll review for your exam, and then we’ll start by revisiting Exponentials and Logarithms.

The general goals for the section **Exponentials and Logarithms** are as follows. At the end of this section, students should be able to:

- Graph exponential functions and identify their major features.
- Understand the significance of exponential functions and their major differences from other types of functions.
- Solve equations with exponentials and logarithms.
- Use exponential and logarithmic functions to model real-world phenomena.

More specifically, at the end of this week you should be able to:

- State the rules of logarithms.
- Use rules of logarithms to rewrite expressions.
- Use the change of base formula to rewrite expressions and compute logarithms.
- Identify an exponential function (formula) based on its graph.
- Describe the difference between exponential growth and linear or polynomial growth.
- Solve equations involving logarithms and exponentials.

Sample Problems. Here are some sample problems, of the type that you would do to demonstrate that you’ve learned the material. These are not the only types of problems you may see – they’re just a sample.

- Write \( \ln(x) + 2 \ln(z) - \ln(4) \) as a single logarithm.
- Expand \( \ln\left(\frac{1}{2x}\right) \) and simplify as much as possible.
- Let’s say \( \log_3(4) = a \) and \( \log_5(3) = b \) and \( \log_4(3) = c \) and \( \log_4(5) = d \). What is \( \log_5(4) \)?
- Sketch a graph of \( y = \frac{1}{2} x - 4 \).
- Which is a bigger number, 100 to the 200th power or 200 to the 100th power?
- Solve for \( x \): \( 4 \cdot 16^{3x} = 16^{x-2} \).
- Simplify \( e^{5 \ln(3)} \) as much as possible.
- Say you know that \( \ln(a) = 5 \). What can you say about \( \ln(ae^3) \)?
- Here’s a graph of an exponential function of the form \( f(x) = a^x + b \) (graph omitted). What is its base, and what is \( b \)?
- Given these two graphs without labels (graphs omitted), which is the graph of \( e^x \) and which is the graph of \( 3^x \)?
- If \( f(x) = \frac{1}{10^x} \), then as \( x \to \infty \), \( f(x) \to \)?
- Does the function \( g(x) = \frac{1}{x^2} \) have any horizontal asymptotes? Explain.
- Let \( f(x) = x^3 \) and \( g(x) = 3^x \). Order the following quantities from smallest to largest: \( f(1.849), f(1849), g(1.849), g(1849) \).
- Is an exponential function always 1-1?
- Graph \( e^x, x^e, \) and \( e^x \).