Solving Trigonometric Equations

The easiest trig equations just involve a good knowledge of the unit circle.

1. Find a value for $x$ such that $\sin(x) = -\frac{\sqrt{2}}{2}$.

2. Find a value for $\theta$ such that $\cos(\theta) = \frac{1}{2}$.

3. Find a value for $t$ such that $\tan(t) = -\sqrt{3}$.

In the above, you found a solution to those equations. When dealing with trig functions, however, there may be more than one solution. In fact, there’s usually an infinite number of solutions. Given an angle $\theta$, we can write all angles that are coterminal with $\theta$ as “$\theta + 2\pi k$, for any integer $k$.” For example, if we want to represent the set of angles $\{0, 2\pi, 4\pi, 6\pi, -2\pi, -4\pi, \ldots\}$, we could just write “$0 + 2\pi k, k \in \mathbb{Z}$” (that “$k \in \mathbb{Z}$” stuff is mathematician shorthand for “$k$ is any integer.”).

4. Find all values of $x$ such that $\sin(x) = -\frac{\sqrt{2}}{2}$.

5. Find all values of $t$ such that $\tan(t) = 1$.

6. Find all values of $\theta$ such that $\csc(\theta) = 1$.

If you have a more complicated trig equation, your main goal is to use algebraic techniques to transform it into something simple, like one of those above.

7. Solve for $t$: $\sqrt{2}\cos t = -1$. 
8. Solve for $t$: \[ \frac{3 + 2 \sin t}{5} = \sin t. \]

Sometimes we get tired of writing $+2\pi k$ all the time. A common thing to do is to restrict our attention to solutions that lie in the interval $[0, 2\pi)$.

9. Find all solutions in the interval $[0, 2\pi)$: \[ 1 = 1 + \frac{3 \cos \theta}{5 \cos \theta - 2}. \]

10. Find all solutions in the interval $[0, 2\pi)$: \[ \frac{6 \sec t + 2}{2 \sec t - 1} = 2. \]

Sometimes, some more complicated algebraic techniques might be required. Things like factoring, and then using the fact that $AB = 0 \implies A = 0$ or $B = 0$. Things like using the fact that $\sec(x) = \frac{1}{\cos(x)}$, or $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Things like treating $\sin(x)$ as a single “thing” (which it is), and factoring $\sin^2(x) - 2 \sin(x) - 3$ exactly the same way you would factor $u^2 - 2u - 3$.

11. Find all solutions in $[0, 2\pi)$: \[ 2 \sin^2 t + \sqrt{3} \sin t = 0 \text{ (Try factoring the left hand side.)} \]
12. Find all solutions: $2 \sin t \cos t = \sin t$ (Try moving all terms to one side and then factoring.)

13. Find all solutions in $[0, 2\pi)$: $2 \cos^2 t + \cos t - 1 = 0$. (Try factoring it like a quadratic.)

14. Find all solutions in $[0, 2\pi)$: $\sin t + \tan t = 0$. (Try rewriting $\tan(x)$, then factoring.)

15. Solve for $\theta$: $2 \sin^2 \theta - 3 \sin \theta + 1 = 0$

16. Solve for $x$: $\tan x \sec x + \sqrt{2} \tan x = 0$
Sometimes your answers have to be expressed using inverse trig functions, since they won’t always work out nicely.

17. Find two solutions for $x$: $3 \cos^2(x) + \cos(x) - 2 = 0$.

What if you had a more complicated expression inside a trig function? Something like $\tan(2x)$? Hint: Let $u = 2x$, solve for $u$, and then substitute back to solve for $x$.

18. Find all solutions in $[0, 2\pi)$: $\tan(\frac{x}{2}) = 1$.

19. Find all solutions: $\cos(2x) = -\frac{\sqrt{2}}{2}$.
You can also use trig identities to help out with simplifying equations.

20. Find all solutions to $\sin(2x) = \cos x$.

21. Solve $\sec^2 x - 2 \tan x = 4$.

22. Find all solutions in $[0, 2\pi)$ of $2 \cot^2(x) + \csc^2(x) - 2 = 0$. 