MATH 3631 - Actuarial Mathematics II
Spring 2013 - Valdez
Homework No. 5
due Monday, 7:00 PM, April 15, 2013

Revised 4/20/2013

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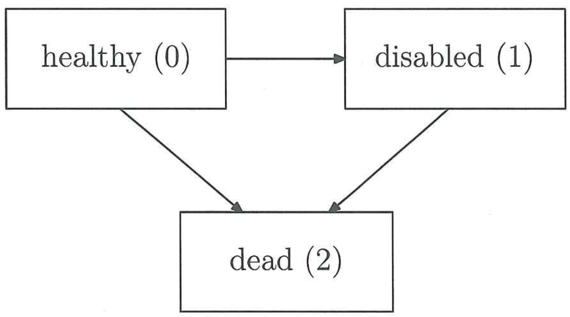
A special permanent disability policy is being priced using a multiple state model with states as expressed in the following diagram:



You are given:

- The policy is issued to a healthy person age x.
- The forces of transitions are independent of age and time:

$$\mu^{01} = 0.001 \quad \mu^{02} = 0.005 \quad \mu^{12} = 0.012$$

- For the next 10 years, the death benefit is \$100,000 for a healthy policyholder and \$50,000 for a disabled policyholder. No death benefit is payable after 10 years from issue.
- For the next 10 years, the disability benefit is payable continuously at the rate of \$25,000 per year. No disability benefit is payable after 10 years from issue.
- Premiums are payable continuously at the rate of P per year while policyholder is healthy, for a maximum of 10 years.
- $\delta = 5\%$

(a) (3 points) Calculate ${}_{10}p_x^{00}$, ${}_{10}p_x^{01}$, and ${}_{10}p_x^{02}$.

(b) (4 points) Calculate P based on the equivalence principle.

(c) (3 points) Calculate the reduction in P if there is no death benefit associated with a disabled policyholder.

For any $t > 0$,

$$0 \longrightarrow 0,2 \longrightarrow 2$$
$$0 \longrightarrow 0,1 \longrightarrow 1$$
$$0 \qquad s \qquad t$$

$${}_tp_x^{00} = e^{-.006t} \qquad e^{-.012(t-s)}$$

$$\qquad \qquad \qquad \qquad \uparrow \; =$$

$${}_tp_x^{01} = \int_0^t e^{-.006s}\, .001 \cdot {}_{t-s}p_{x+s}^{11}\, ds = e^{-.012t}\frac{.001}{.006}\left(e^{.006t}-1\right)$$

$$= \frac{1}{6}\left(e^{-.006t} - e^{-.012t}\right)$$

${}_tp_x^{02}$ is best determined by noting that ${}_tp_x^{00} + {}_tp_x^{01} + {}_tp_x^{02} = 1$

$1 - {}_tp_x^{00} - {}_tp_x^{01}$ (Note: see last page for doing this from first principle)

(a) set $t = 10$

$${}_{10}p_x^{00} = e^{-.006(10)} = .9417645$$

$${}_{10}p_x^{01} = \frac{1}{6}\left(e^{-.006(10)} - e^{-.012(10)}\right) = .009140683$$

$${}_{10}p_x^{02} = 1 - .9417645 - .009140683 = .04909478$$

(b) $$APV(FP_0) = P\int_0^{10} e^{-.05t}\,{}_tp_x^{00}\,dt = P\int_0^{1} e^{-.056t}\,dt$$

$$= P\frac{1}{.056}\left(1-e^{-.056}\right) = 7.656981\,P$$

$$APV(FDB_0) = \int_0^{10} e^{-.05t}\cdot 50{,}000\left(2\,\underbrace{{}_tp_x^{00}}_{e^{-.006t}}\,\underbrace{\mu^{02}}_{.005} + \underbrace{{}_tp_x^{01}}_{\frac{1}{6}\left(e^{-.006t}-e^{-.012t}\right)}\,\underbrace{\mu^{12}}_{.012}\right)dt$$

$$\underbrace{= 50{,}000\left[\frac{2(.005)}{.056}\left(1-e^{-.056(10)}\right) + \frac{.012}{6}\left(\frac{1}{.056}\left(1-e^{-.056(10)}\right) - \frac{1}{.062}\left(1-e^{-.062(10)}\right)\right)\right]}$$

$$= 3848.938$$

$$\text{APV}(\overset{\text{disability}}{\text{FDiBo}}) = \int_0^{10} e^{-.05t} \, {}_tp_x^{01} \, dt * 25000$$

$$= 25000 \int_0^{10} e^{-.05t} \, \tfrac{1}{6}\left(e^{-.006t} - e^{-.012t}\right) dt$$

$$= \underbrace{\frac{25000}{6}\left[\frac{1}{.056}\left(1-e^{-.056(10)}\right) - \frac{1}{.062}\left(1-e^{-.062(10)}\right)\right]}_{= 851.9664}$$

So that ~~7.656981~~ $P = \dfrac{3848.938 + 851.9664}{7.656981}$

$\Rightarrow$ $\underline{\underline{P = 613.9371}}$

(c) If no death benefit for disabled policyholders, then

$$\text{APV}(\text{FDB}_0) = 100{,}000 \int_0^{10} e^{-.05t} \, \underbrace{{}_tp_x^{00}}_{e^{-.006}} \, \underbrace{\mu^{02}}_{.005} \, dt$$

$$= 100{,}000 \left(\frac{.005}{.056}\right)\left(1-e^{-.056(10)}\right) = 3828.491$$

so that the new premium rate is

$$P^{new} = \frac{3828.491 + 851.9664}{7.656981} = \underline{\underline{611.2667}}$$

a slim reduction of $\dfrac{613.9371 - 611.2667}{613.9371} < 1\%$

The death benefit associated with ~~sick~~ disabled policyholders provides, interestingly, very little value to the policy.

Notice that death from 0 can be in one of 2 ways: $0 \to 2$ or $0 \to 1 \to 2$ Then

$0 \longrightarrow 1 \longrightarrow 2$

$0 \longrightarrow 2$

$0 \quad s \quad t$

$$
{}_tp_x^{02} = \int_0^t e^{-.006s}\,.005\,ds
$$

$$
+ \int_0^t {}_sp_x^{00}\,\mu^{01}\underbrace{\left(1 - {}_{t-s}p_{x+t}^{11}\right)}\,ds
$$

prob that you do not end up in 1 but in 2 instead

~~this is also~~

$$
= \frac{5}{6}\left(1 - e^{-.006}\right) + \underbrace{\int_0^t e^{-.006s}\,.001\,ds}_{\frac{1}{6}\left(1 - e^{-.006}\right)} - {}_tp_x^{01}
$$

$$
= 1 - e^{-.006} - {}_tp_x^{01}
$$

$$
= 1 - {}_tp_x^{00} - {}_tp_x^{01}
$$