

MATH 3631 - Actuarial Mathematics II  
 Spring 2010 - Valdez  
 Homework No. 4  
 due Monday, 4:50 PM, 29 March 2010

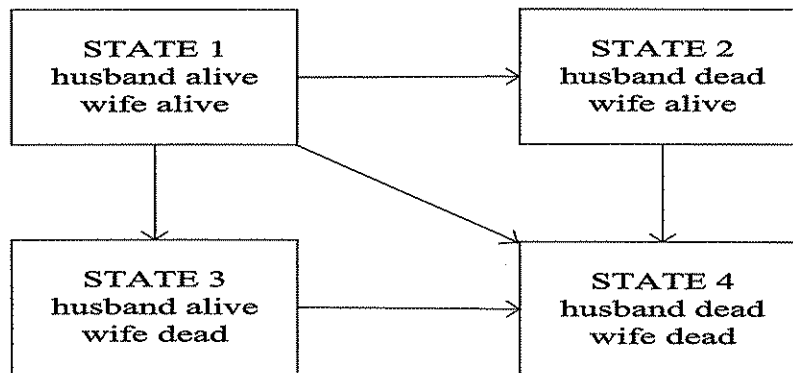
Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: EMIL Student ID: SUGGESTED SOLUTION

I certify that this is my own work, and that I have not copied the work of another student.

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A four-state homogeneous Markov model represents the joint mortality of a married couple: a husband and a wife. The states are: 1 = husband alive, wife alive; 2 = husband dead, wife alive; 3 = husband alive, wife dead, and 4 = both husband and wife dead.



The one-year transition probabilities are:

$$\begin{matrix}
 & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\
 \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 0.95 & 0.02 & 0.02 & 0.01 \\ 0.00 & 0.95 & 0.00 & 0.05 \\ 0.00 & 0.00 & 0.94 & 0.06 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{pmatrix}
 \end{matrix}$$

A life insurer sells a two-year term insurance contract to a married couple who are both age 60. The death benefit of 100,000 is payable at the end of the year in which the second life dies, if both die within 2 years.

Premiums are payable annually in advance. Interest rate  $i = 6\%$ .

Calculate the annual benefit premium.

It is a two-year term policy



HW4

Assume premium  $P$  (annual) is payable at beginning of year so long as at least one is alive.

<u>Year</u>	<u>transitions</u>	<u>probability</u>	<u>amount</u>	<u>A.P.V.</u>
0	-	1	$P$	$P$
1	$1 \rightarrow 1$	.95	$P$	$.99 P v = .934 P$
	$1 \rightarrow 2$	.02		
	$1 \rightarrow 3$	.02		
				$\Sigma = 1.934 P$

Benefits (only at end of year, 2nd death)

<u>Year</u>	<u>transitions</u>	<u>probability</u>	<u>amount</u>	<u>A.P.V.</u>
1	$1 \rightarrow 4$	.01	100,000	$.01 \times 100,000 v$ $= 943.40$
2	$1 \rightarrow 1 \rightarrow 4$	$.95(.01) = .0095$	100,000	$.0095 \times 100,000 v^2$ $= 845.50$
	$1 \rightarrow 2 \rightarrow 4$	$.02(.05) = .001$		$.001 \times 100,000 v^2$ $= 89.00$
	$1 \rightarrow 3 \rightarrow 4$	$.02(.06) = .0012$		$.0012 \times 100,000 v^2$ $= 106.80$
				$\Sigma = 1984.70$

$APV(\text{Premiums}) = APV(\text{Benefits})$

$1.934 P = 1984.70$

$P = \frac{1984.70}{1.934} = 1,026.22$  is the annual benefit premium