

MATH 3631 - Actuarial Mathematics II
Spring 2016 - Valdez
Homework No. 3
due Monday, 5:00 PM, 21 March 2016

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: Suggested Solution Student ID: EMIL

I certify that this is my own work, and that I have not copied the work of another student.

Signature: _____ Date: _____

An insurer issued 500,000 fully discrete whole life insurance policies to lives all exactly age 50 on January 1, 2005. Each policy issued has a death benefit of \$150,000 with an annual gross premium of \$3,875.

You are given:

- The following values in Year 2014:

	anticipated	actual
Expenses as a percent of premium	0.04	0.05
Annual effective rate of interest	4.0%	6.2%
q_{59}	0.0080	0.0095

- The gross premium reserve per policy at the end of Year 2013 is ${}_9V^g = 3,066.48$.
 - A total of 471,748 remain in force at the beginning of Year 2014.
- Calculate the gross premium reserve per policy at the end of Year 2014.
 - Calculate the total gain (or loss) for this portfolio of policies in Year 2014.
 - Calculate the gain (or loss) by source emerging at the end of Year 2014 using the following order: interest then expenses then mortality.
 - Calculate the gain (or loss) by source emerging at the end of Year 2014 using the following order: mortality then interest then expenses.
 - Show that the sums of the gain (or loss) in each of (3) and (4) above both equal to that in (2). Is there a difference in the gain (or loss) due to mortality? Explain why.

HW3

(P.1)

$$\textcircled{1} \quad {}_{10}V^g = \frac{(3066.48 + 3875(1-.04))(1.04) - 150000(.0080)}{1-.0080}$$

$$= 5905,181$$

② Total gain (or loss)

$$\text{Actual Reserve} = 471748 \left[\frac{(3066.48 + 3875(1-.05))(1.062) - (150000 - 5905.181)(.0095)}{1} \right]$$

$$= 2,734,812,069$$

$$\text{Expected Reserve} = 471748 (5905.181) \text{ OR}$$

$$471748 \left[\frac{(3066.48 + 3875(1-.04))(1.04) - (150000 - 5905.181)(.0080)}{1} \right]$$

$$= 2,785,757,159$$

Actual - Expected = -50,945,090, a loss of about \$1 million!

③ Gain (or loss) by source:

$$\text{interest: } \underbrace{471748 [3066.48 + 3875(.96)]}_{+70,433,184} \cdot (.062 - .04)$$

a gain since interest actually earned is higher than anticipated

$$\text{expenses: } \underbrace{471748 [3875(.04 - .05)(1.062)]}_{-19,413,610}$$

a loss since actual expenses exceed expected expenses

mortality: $471748 (150000 - 5905.181)(.0080 - .0095)$ (1.2)

$-101,964,664$ a loss since more deaths than anticipated

④ Gain (or loss) by source:

mortality: same as in ③ since mortality gain/loss is unaffected by either interest or expense

interest: same as in ③ since unaffected by mortality

expenses: same as in ③ since unaffected by mortality but similarly affected by interest

⑤ Adding the components of ③ or ④, we get

$$\begin{aligned} &+70,433,184 - 19,413,610 - 101,964,664 \\ &= -50,945,090 \end{aligned}$$

Note that the large loss was mainly due to deviation in mortality. There is no difference in the loss due to mortality, whether you accounted for it first or last, because mortality is not affected by either interest or expenses, the other sources.

However, if there were expenses associated with death, such would not have been true!