MATH 3631 - Actuarial Mathematics II
Spring 2013 - Valdez
Homework No. 3
due Wednesday, 7:00 PM, March 13, 2013

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: SUGGESTED SOLUTIONS **Student ID:** _____

I certify that this is my own work, and that I have not copied the work of another student.

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For a life insurance issued to (35), you are given:

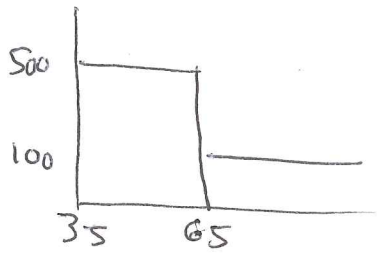
- The death benefit, payable at the end of the year of death, is equal to \$500 up to age 65 and \$100 thereafter.
- Benefit premiums are level, payable annually, and only for the first 30 years. [1]
- The accumulation of all benefit premiums paid, without interest, is refunded at age 65 if the insured is still alive then.
- Mortality follows the Illustrative Life Table with $i = 6\%$.
- Deaths are uniformly distributed over each year of age.

(a) (2.5 points) Calculate the annual benefit premium.

(b) (2.5 points) Calculate the benefit reserve at the end of 10 years.

(c) (2.5 points) Calculate the benefit reserve at the end of 10.75 years.

(d) (2.5 points) Calculate the benefit reserve at the end of 40 years.



(a) Let P = annual benefit premium

$$APVFB_0 = 500\,A_{35} - 400\,{}_{30}E_{35}\,A_{65} + 30P\,{}_{30}E_{35}$$

$$APVFP_0 = P\,\ddot{a}_{35:\overline{30}|}$$

[1] corrected on 2 March 2013

From the table, we have ${}_{30}E_{35} = {}_{20}E_{35}\ {}_{10}E_{55} = (.28600)(.48686)$
$= .139242$

$\ddot{a}_{35} = 15.3926$ $\quad \ddot{a}_{65} = 9.8969$

$A_{35} = .12872$ $\quad A_{65} = .43980$

$\ddot{a}_{35:\overline{30}|} = \ddot{a}_{35} - {}_{30}E_{35}\,\ddot{a}_{65} = 15.3926 - .139242\,(9.8969)$
$= 14.01454$

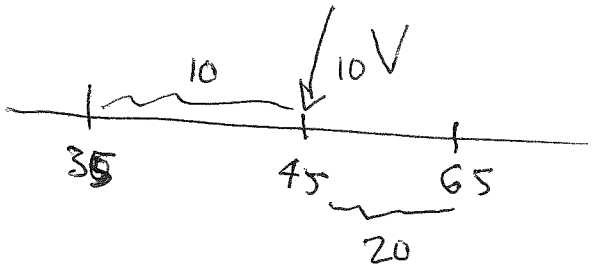
Set $APVFP_0 = APVFB_0$ and solve for P, we get

$$P(\ddot{a}_{35:\overline{30}|} - 30\ {}_{30}E_{35}) = 500A_{35} - 400\ {}_{30}E_{35}\,A_{65}$$

$$P = \frac{500(.12872) - 400(.139242)(.43980)}{14.01454 - 30(.139242)}$$

$$= (39.86455 / 9.837277) = \underline{4.052397}$$

(b)



$10V = APVFB_{10} - APVFP_{10}$

$= 500A_{45} - 400\ {}_{20}E_{45}\,A_{65} + 30P\ {}_{20}E_{45} - P\,\ddot{a}_{45:\overline{20}|}$

From the table, $A_{45} = .20120$ $\quad {}_{20}E_{45} = .25634$

$\ddot{a}_{45} = 14.1121$

$\ddot{a}_{45:\overline{20}|} = \ddot{a}_{45} - {}_{20}E_{45}\,\ddot{a}_{65} = 14.1121 - (.25634)(9.8969)$
$= 11.57513$

$$APVFB_{10} = \underbrace{500(.20120) - 400(.25634)(.43980) + 30(4.052397)(.25634)}_{= 86.66841}$$

$$APVFP_{10} = (4.052397)(11.57513) = 46.90702$$

$$10V = 86.66841 - 46.90702 = \underline{\underline{39.76139}}$$

(c)

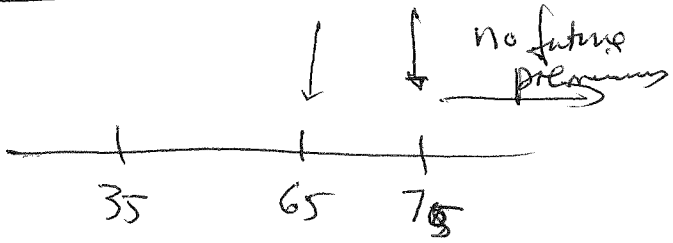
$$10.75V = \frac{(10V + P)(1.06)^{.75} - 500\, v^{.25}\, .75\, {}_{.75}q_{45}}{1 - .75\, {}_{.75}q_{45}}$$

(4/1000)

$$= \frac{(39.76139 + 4.052397)(1.06)^{.75} - 500(1.06)^{-.25}(.75)(.004)}{1 - .75(.004)}$$

$$= \frac{44.29267}{0.997} = \underline{\underline{44.42595}}$$

(d)



$$40V = APVFB_{40}$$

$$= 100\, A_{75}$$

$$= 100(.59149) = \underline{\underline{59.149}}$$