MATH 3631 - Actuarial Mathematics II
Spring 2016 - Valdez
Homework No. 2
due Monday, 5:00 PM, 15 February 2016

Please return this page with your signature. Please write your name and student number at the spaces provided:

Name: Suggested Solutions **Student ID:** ____________

I certify that this is my own work, and that I have not copied the work of another student.

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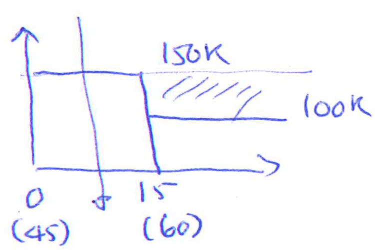
For a whole life insurance issued to (45), you are given:

- The death benefit, payable at the moment of death, is \$150,000 in the first 15 years and \$100,000 thereafter.
- Benefit premiums, payable at the beginning of each year, are level throughout the contract.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$
- Deaths are uniformly distributed over each year of age.

1. Calculate the net premium reserve at the end of 10 years.
2. Calculate the net premium reserve at the end of 10.6 years.

Now suppose expenses, payable at the beginning of each year, consist of: (a) 8% of the gross annual premium in the first year and (b) 3% of the gross annual premium in subsequent years.

3. Calculate the gross premium reserve at the end of 10 years.
4. Calculate the gross premium reserve at the end of 10.6 years.



First calculate the net annual premium:

$$P = \frac{50{,}000\left[3\bar{A}_{45} - {}_{15}E_{45}\bar{A}_{60}\right]}{\ddot{a}_{45}}$$

$$= \frac{50{,}000\left[3A_{45} - {}_{15}E_{45}A_{60}\right]\frac{i}{\delta}}{\ddot{a}_{45}}$$

$i = .06$, $\delta = \log(1.06)$

$A_{45} = .20120$

$A_{60} = .36913$

${}_{15}E_{45} = {}_{10}E_{45}\,{}_{5}E_{55} = .52652(.70810)$

$\ddot{a}_{45} = 14.1121$

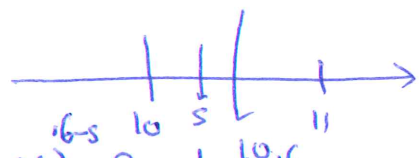
$$P = 50{,}000\left[3(.20120) - .52652(.70810)(.36913)\right]\frac{.06}{\log(1.06)} \Big/ 14.1121$$

$$= \frac{23991.06}{14.1121} = \underline{\underline{1{,}700.035}}$$

(1) $${}_{10}V^n = APV(FB_{10}) - APV(FP_{10})$$

$$= 50{,}000\left[3\underbrace{A_{55}}_{.30514} - \underbrace{{}_5E_{55}}_{.70810}\underbrace{A_{60}}_{.36913}\right]\frac{i}{\delta} - P\,\overbrace{\ddot{a}_{55}}^{12.2758}$$

$$= \underline{\underline{12{,}804.19}}$$



(2) $${}_{10.6}V^n = \frac{({}_{10}V + P)(1.06)^{.6} - 150{,}000\int_0^{.6}(1.06)^{.6-s}\cdot q_{55}\,ds}{1 - \int_0^{.6} q_{55}\,ds}$$

$$= \frac{(\overset{12804.19}{{}_{10}V} + \overset{1700.035}{P})(1.06)^{.6} - 150000\,\dfrac{e^{.6\delta}-1}{\delta}\,\dfrac{8.96}{1000}}{1 - .6\left(\dfrac{8.96}{1000}\right)}$$

$$= \frac{14199.62}{0.994624} = 14{,}276.37$$

Next, compute the gross annual premium

$$G\ddot{a}_{45} = 50{,}000\left[3A_{45} - {}_{15}E_{45}A_{60}\right]i/\delta + .03G\ddot{a}_{45} + .05G$$

$$G = \frac{50{,}000\left[3A_{45} - {}_{15}E_{45}A_{60}\right]i/\delta}{.97\ddot{a}_{45} - .05} = \frac{23991.06}{13.63874} = 1759.039$$

(3) $${}_{10}V^g = APV(FB_{10}) + APV(FE_{10}) - APV(FP_{10})$$

$$= 50{,}000\Big[3\underset{.30514}{A_{55}} - \underset{.70810}{{}_5E_{45}}\,\underset{.36913}{A_{60}}\Big]\frac{i}{\delta} + \underbrace{.03G\ddot{a}_{55} - G\ddot{a}_{55}}_{-.97G\underset{12.2758}{\ddot{a}_{55}}}$$

$$= \underline{\underline{12{,}727.69}}$$

(4) $${}_{10.6}V^g = \frac{\big({}_{10}V^g + \overbrace{G - .03G}^{+.97G}\big)(1.06)^{.6} - 150{,}000\,\frac{e^{.6\delta}-1}{\delta}\,\frac{8.96}{1000}}{1 - .6\left(8.96/1000\right)}$$

$$= \frac{14126.85}{0.994624} = \underline{\underline{14{,}203{,}20}}$$