Exercise 9.4

(a) $\ddot{a}_{25:30} = \sum_{k=0}^{\infty} v^k \, {}_kp_{25} \, {}_kp_{30} = 15.89014$

```
A <- 0.0001
B <- 0.0003
c <- 1.075
muxt <- function(x,t){
A + B*c^(x+t)}
tpx <- function(x,t){
temp <- A*t + B*c^x*(c^t-1)/log(c)
exp(-temp)}
i <- 0.05
v <- 1/(1+i)
# limiting age
w <- 131
tt <- max(w-25,w-31)
t <- 0:tt
vt <- v^t
ann2530 <- sum(vt*tpx(25,t)*tpx(30,t))

> ann2530
[1] 15.89014
```

(b) $\ddot{a}_{\overline{25:30}} = \ddot{a}_{25} + \ddot{a}_{30} - \ddot{a}_{25:30} = 17.76571 + 17.09146 - 15.89014 = 18.96704$

```
ann25 <- sum(vt*tpx(25,t))
ann30 <- sum(vt*tpx(30,t))
ann2530B <- ann25 + ann30 - ann2530
ann2530B

> ann25
[1] 17.76571
> ann30
[1] 17.09146
> ann2530B
[1] 18.96704
```

(c) $\ddot{a}_{25|30} = \ddot{a}_{30} - \ddot{a}_{25:30} = 17.09146 - 15.89014 = 1.201325$

(d) $\bar{A}_{25:30} = \int_0^{\infty} v^t \, {}_tp_{25:30}\left(\mu_{25+t} + \mu_{30+t}\right)dt = 0.2493299$

```
h <- 1/1000
t <- seq(0,tt,h)
vt <- v^t
```

```
intA <- vt*tpx(25,t)*tpx(30,t)*(muxt(25,t)+muxt(30,t))
Ab2530 <- 0
n <- 1
while (n<length(t)) {
n <- n+2
Ab2530 <- Ab2530 + (h/3)*(intA[n-2]+4*intA[n-1]+intA[n])
}

> Ab2530
[1] 0.2493299
```

(e) $\bar{A}^{\,1}_{25:30:\overline{10}|} = \int_0^{10} v^t \,_tp_{25}\,\mu_{25+t} \cdot \,_tp_{30}\,dt = 0.02076931$

```
t <- seq(0,10,h)
vt <- v^t
intA2 <- vt*tpx(25,t)*tpx(30,t)*muxt(25,t)
A2530term10 <- 0
n <- 1
while (n<length(t)) {
n <- n+2
A2530term10 <- A2530term10 + (h/3)*(intA2[n-2]+4*intA2[n-1]+intA2[n])
}

> A2530term10
[1] 0.02076931
```

(f) $\bar{A}^{\,2}_{25:30} = \int_0^{\infty} v^t \,_tp_{30}\,\mu_{30+t}\bigl(1 - \,_tp_{25}\bigr)\,dt = 0.04402224$

```
t <- seq(0,tt,h)
vt <- v^t
intA3 <- vt*tpx(30,t)*muxt(30,t)*(1-tpx(25,t))
A25302 <- 0
n <- 1
while (n<length(t)) {
n <- n+2
A25302 <- A25302 + (h/3)*(intA3[n-2]+4*intA3[n-1]+intA3[n])
}

> A25302
[1] 0.04402224
```