

Exercise 9.14

For lives (x) and (y) with independent future lifetimes, we have

$$\begin{aligned}\text{Cov}[v^{T_{xy}}, v^{T_{\overline{xy}}}] &= \text{E}[v^{T_{xy}} \cdot v^{T_{\overline{xy}}}] - \text{E}[v^{T_{xy}}]\text{E}[v^{T_{\overline{xy}}}] \\ &= \text{E}[v^{T_x} \cdot v^{T_y}] - \text{E}[v^{T_{xy}}]\text{E}[v^{T_{\overline{xy}}}] \\ &= \text{E}[v^{T_x}]\text{E}[v^{T_y}] - \text{E}[v^{T_{xy}}]\text{E}[v^{T_{\overline{xy}}}] \\ &= \bar{A}_x \bar{A}_y - \bar{A}_{\overline{xy}} \bar{A}_{xy} \\ &= \bar{A}_x \bar{A}_y - (\bar{A}_x + \bar{A}_x - \bar{A}_{xy}) \bar{A}_{xy} \\ &= \bar{A}_x \bar{A}_y - \bar{A}_x \bar{A}_{xy} - \bar{A}_y \bar{A}_{xy} + (\bar{A}_{xy})^2 \\ &= (\bar{A}_x - \bar{A}_{xy})(\bar{A}_y - \bar{A}_{xy})\end{aligned}$$