Exercise 9.11

The APV of future benefits at issue is equal to

$$\text{APV}(\text{FB}_0) = 100000\,\bar{A}_{28:24} = 100000\int_0^\infty v^t\,{}_tp^m_{28}\,{}_tp^f_{24}\big(\mu^m_{28+t}+\mu^f_{24+t}\big)dt,$$

where the superscripts m and f refer to the husband and wife, respectively. Let G denote the annual gross premium so that the APV of future premiums is equal to

$$\text{APV}(\text{FP}_0) = G\,\ddot{a}^{(12)}_{28:24:\overline{25|}}.$$

Finally, the APV of future expenses at issue is equal to

$$\text{APV}(\text{FE}_0) = 250 - 0.03(G/12) + 0.03G\,\ddot{a}^{(12)}_{28:24:\overline{25|}},$$

where here $\ddot{a}^{(12)}_{28:24:\overline{25|}}$ denotes the APV of an annuity of \$1 per year payable monthly so long as both (28) and (24) are alive, up to 25 years:

$$\ddot{a}^{(12)}_{28:24:\overline{25|}} = \frac{1}{12}\sum_{k=0}^{25-(1/2)} v^{k/12}\,{}_{k/12}p^m_{28}\,{}_{k/12}p^f_{24}$$

Solving for G, we get

$$\begin{aligned} G &= \frac{100000\,\bar{A}_{28:24}+250}{0.97\,\ddot{a}^{(12)}_{28:24:\overline{25|}}+(0.03/12)} \\ &= \frac{100000(0.2484613)+250}{0.97(13.32663)+(0.03/12)} \\ &= 1941.024 \end{aligned}$$

so that the monthly premium is

$$G/12 = 1941.024/12 = 161.752.$$

```
mu28m <- function(t){
A <- 0.0001
B <- 0.0004
c <- 1.075
A + B*c^(28+t)}
tp28m <- function(t){
A <- 0.0001
B <- 0.0004
c <- 1.075
temp <- A*t + B*c^28*(c^t-1)/log(c)
exp(-temp)}
mu24f <- function(t){
A <- 0.0001
B <- 0.0003
```

```
c <- 1.07
A + B*c^(24+t)}
tp24f <- function(t){
A <- 0.0001
B <- 0.0003
c <- 1.07
temp <- A*t + B*c^24*(c^t-1)/log(c)
exp(-temp)}
i <- 0.05
v <- 1/(1+i)
h <- 1/1000
# limiting age
w <- 150
tt <- max(w-28,w-24)
t <- seq(0,tt,h)
vt <- v^t
intA <- vt*tp28m(t)*tp24f(t)*(mu28m(t)+mu24f(t))
apvfb <- 0
n <- 1
while (n<length(t)) {
n <- n+2
apvfb <- apvfb + (h/3)*(intA[n-2]+4*intA[n-1]+intA[n])
}
k <- seq(0,25-(1/12),1/12)
vk <- v^k
ann2824temp25 <- (1/12)*sum(vk*tp28m(k)*tp24f(k))
num <- 100000*apvfb + 250
den <- 0.97*ann2824temp25 + .03*(1/12)
G <- num/den

> apvfb
[1] 0.2484613
> ann2824temp25
[1] 13.32663
> G
[1] 1941.024
> G/12
[1] 161.752
```

Note the slight difference in the answer from that in the published text. This is because the book included a 3% of the first month's premium for expenses, as indicated below:

$$\frac{G}{12} = \frac{100000\,\bar{A}_{28:24} + 250}{0.97\,\ddot{a}^{(12)}_{28:24:\overline{25}|}} = \frac{100000(0.2484613) + 250}{0.97(13.32663)} = 161.7833$$

This is misleading because a renewal expense by definition does not include expense in the first premium.