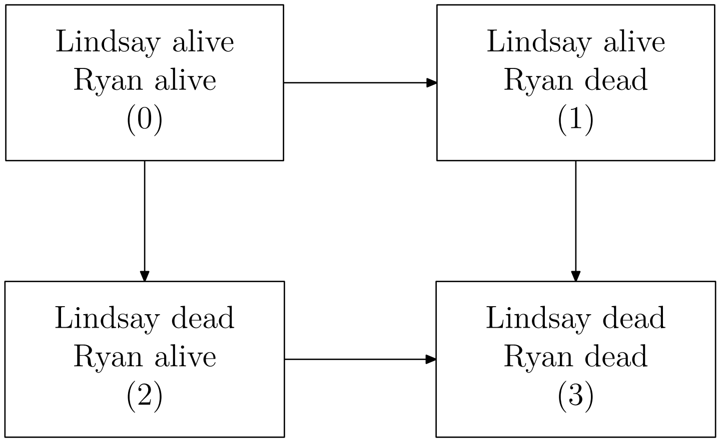
Exercise 9.10

First, we note that this problem can be depicted as a joint life model as shown in the figure below:



(a) The APV of Ryan's retirement benefits at his retirement date is given by

$$\text{APV(FB)} = \frac{1}{12} 100000 \sum_{k=0}^{\infty} v^{k/12} {}_{k/12}p_{65}^{R} = 802639.3.$$

```
mu65tR <- function(t){
A <- 0.0001
B <- 0.0004
c <- 1.075
A + B*c^(65+t)}
tpxR <- function(x,t){
A <- 0.0001
B <- 0.0004
c <- 1.075
temp <- A*t + B*c^x*(c^t-1)/log(c)
exp(-temp)}
mu63tL0 <- function(t){
A <- 0.0001
B <- 0.00025
c <- 1.07
A + B*c^(63+t)}
tpxL0 <- function(x,t){
A <- 0.0001
B <- 0.00025
c <- 1.07
temp <- A*t + B*c^x*(c^t-1)/log(c)
exp(-temp)}
mu63tL1 <- function(t){
A <- 0.0001
B <- 0.0003
```

```
c <- 1.072
A + B*c^(63+t)}
tpxL1 <- function(x,t){
A <- 0.0001
B <- 0.0003
c <- 1.072
temp <- A*t + B*c^x*(c^t-1)/log(c)
exp(-temp)}
i <- 0.05
v <- 1/(1+i)
# limiting age
w <- 150
tt <- w-65
t <- seq(0,tt,1/12)
vt <- v^t
ann65R <- (100000/12)*sum(vt*tpxR(65,t))

> ann65R
[1] 802639.3
```

(b) Let B denote the monthly benefit while both are alive. When both are alive, the APV of future benefits is given by

$$B \times \sum_{k=0}^{\infty} v^{k/12} {}_{k/12}p_{65}^{R} \, {}_{k/12}p_{63}^{L,0} = B \times 82.35144.$$

After Ryan's death, the benefit is reduced to 60% so that the APV of future benefits in this case is given by

$$0.60B \times \int_0^{\infty} v^t \, {}_tp_{65}^{R} \, {}_tp_{63}^{L,0} \, \mu_{65+t}^{R} \times \ddot{a}_{63+t}^{(12)L,1} dt = 0.60B \times 48.60911.$$

Finally, although not clearly stated in the problem, it is understood that the same monthly benefit payable to Ryan continues after the death of Lindsay. In this case, the APV of future benefits is given by

$$B \times \int_0^{\infty} v^t \, {}_tp_{65}^{R} \, {}_tp_{63}^{L,0} \, \mu_{63+t}^{L,0} \times \ddot{a}_{65+t}^{(12)R} dt = B \times 14.05924.$$

A few words about the notation used here. The superscripts R and L refer to that of Ryan and Lindsay, respectively. For Lindsay, we follow up the superscript with the state she is in (either in state 0 or 1). For Ryan, this was not necessary since the forces of mortality are the same before and after widowerhood. Adding all three APV's of future benefits and equating this to the amount in part (a), we get the monthly benefit equal to

$$B = \frac{802639.3}{82.35144 + 0.60(48.60911) + 14.05924} = 6391.655$$

which when multiplied by 12 gives the annual amount of 76699.85. [Note: slightly different from the book!!!]

```
ann00 <- sum(vt*tpxR(65,t)*tpxL0(63,t))
h1 <- seq(0,tt,length=10001)
vh <- v^h1
annm63L1 <- function(s){
kk <- w-63-s
k <- seq(0,kk,1/12)
vk <- v^k
sum(vk*tpxL1(63+s,k))}
annmth63L1 <- rep(0,length(h1))
m <- 0
while (m<length(h1)) {
m <- m+1
annmth63L1[m] <- annm63L1(h1[m])
}
integ1 <- vh*tpxR(65,h1)*tpxL0(63,h1)*mu65tR(h1)*annmth63L1
tempR <- 0
n <- 1
while (n<length(h1)) {
n <- n+2
tempR <- tempR + (h1[2]/3)*(integ1[n-2]+4*integ1[n-1]+integ1[n])
}

annm65R <- function(s){
kk <- w-65-s
k <- seq(0,kk,1/12)
vk <- v^k
sum(vk*tpxR(65+s,k))}
annmth65R <- rep(0,length(h1))
m <- 0
while (m<length(h1)) {
m <- m+1
annmth65R[m] <- annm65R(h1[m])
}
integ2 <- vh*tpxR(65,h1)*tpxL0(63,h1)*mu63tL0(h1)*annmth65R
tempL1 <- 0
n <- 1
while (n<length(h1)) {
n <- n+2
tempL1 <- tempL1 + (h1[2]/3)*(integ2[n-2]+4*integ2[n-1]+integ2[n])
}

num <- ann65R
den <- ann00 + 0.6*tempR + tempL1
B <- num/den

> ann00
```

```
[1] 82.35144
> tempR
[1] 48.60911
> tempL1
[1] 14.05924
> B
[1] 6391.655
> 12*B
[1] 76699.85
```

(c) In a 'pop-up', the only thing affected is the APV of the future benefits immediately following the death of Lindsay, which in this case is given by

$$(100000/12) \times \int_0^\infty v^t \, {}_tp_{65}^R \, {}_tp_{63}^{L,0} \, \mu_{63+t}^{L,0} \times \ddot{a}_{65+t}^{(12)R} dt = (100000/12) \times 14.05924 = 117160.3.$$

The revised monthly benefit then is given by

$$B = \frac{802639.3 - 117160.3}{82.35144 + 0.60(48.60911)} = 6146.862$$

which when multiplied by 12 gives the annual amount of 73762.34. [Note: slightly different from the book!!!]

```
tempL1revised <- (100000/12)*tempL1
num <- ann65R - tempL1revised
den <- ann00 + 0.6*tempR
Brevised <- num/den

> Brevised
[1] 6146.862
> 12*Brevised
[1] 73762.34
```