Exercise 8.14

Let G denote the monthly premium for this policy. The APV of future premiums then can be expressed as

$$\text{APV}(\text{FP}_0) = 12G \times \ddot{a}^{(12)}_{50:\overline{10}|}(*).$$

The APV of future benefits is given by

$$\text{APV}(\text{FB}_0) = 100000 \times \bar{A}^{\,1}_{50:\overline{10}|}(*).$$

The APV of future expenses is given by

$$\text{APV}(\text{FE}_0) = 200 + 0.025 \cdot 12G \times \ddot{a}^{(12)}_{50:\overline{10}|}(*).$$

Note for both the insurance and annuity symbols, we followed it up with the symbol $(*)$ to indicate that they are respectively being evaluated with the extra 2% of lapses in the first two years of anniversary. Using the equivalence principle to solve for G, we then get

$$G = \frac{100000 \times \bar{A}^{\,1}_{50:\overline{10}|}(*) + 200}{0.975 \cdot 12 \times \ddot{a}^{(12)}_{50:\overline{10}|}(*)}$$

We evaluate the annuity based on

$$\ddot{a}^{(12)}_{50:\overline{10}|}(*) = \ddot{a}^{(12)}_{50:\overline{1}|} + 0.98 \cdot v p_{50}\, \ddot{a}^{(12)}_{51:\overline{1}|} + (0.98^2) \cdot v^2 p_{50} p_{51}\, \ddot{a}^{(12)}_{52:\overline{8}|}$$

The annuity symbols on the RHS of the formula indicate calculations based on mortality as the only decrement. Similarly, we evaluate the term insurance component based on (including the UDD assumption within each year of age interval):

$$\bar{A}^{\,1}_{50:\overline{10}|}(*) = \frac{i}{\delta} v q_{50} + 0.98 \cdot v p_{50} \times \frac{i}{\delta} v q_{51} + (0.98^2) \cdot v^2 p_{50} p_{51} \times \frac{i}{\delta} A^{\,1}_{52:\overline{8}|}$$

The details of the calculations are summarized in the R code below:

```
mu <- function(x){
A <- 0.0001
B <- 0.0004
c <- 1.075
A + B*c^x}
tpx <- function(x,t){
A <- 0.0001
B <- 0.0004
c <- 1.075
temp <- A*t + B*c^x*(c^t-1)/log(c)
exp(-temp)}
i <- 0.05
delta <- log(1+i)
v <- 1/(1+i)
# evaluate insurance assuming UDD
```

```
apvfb1 <- (i/delta)*v*(1-tpx(50,1))
apvfb2 <- (i/delta)*v*(1-tpx(51,1))
ann528 <- sum(v^(0:7) * tpx(52,0:7))
d <- 1-v
apvfb3 <- (i/delta)*(1 - d*ann528 - v^8 * tpx(52,8))
# for the annuity portion
mk1 <- (0:11)/12
apvfp1 <- (1/12)*sum(v^mk1 * tpx(50,mk1))
apvfp2 <- (1/12)*sum(v^mk1 * tpx(51,mk1))
mk2 <- (0:95)/12
apvfp3 <- (1/12)*sum(v^mk2 * tpx(52,mk2))

A5010 <- apvfb1 + (.98*tpx(50,1)*v)*apvfb2 + (.98^2*tpx(50,2)*v^2)*apvfb3
am5010 <- apvfp1 + (.98*tpx(50,1)*v)*apvfp2 + (.98^2*tpx(50,2)*v^2)*apvfp3
# monthly gross premium G
num <- 100000*A5010 + 200
den <- 12*.975*am5010
G <- num/den

> A5010
[1] 0.147534
> am5010
[1] 7.050648
> G
[1] 181.2697
```

Here, we find that

$$\ddot{a}^{(12)}_{50:\overline{10}|}(*) = 7.050648$$

and

$$\bar{A}^{\,1}_{50:\overline{10}|}(*) = 0.147534$$

so that the monthly premium comes out to be

$$G = \frac{100000 \times 0.147534 + 200}{0.975 \cdot 12 \times 7.050648} = 181.2697.$$

Not sure why the answer does not match the book! Book does not seem to account for expenses.