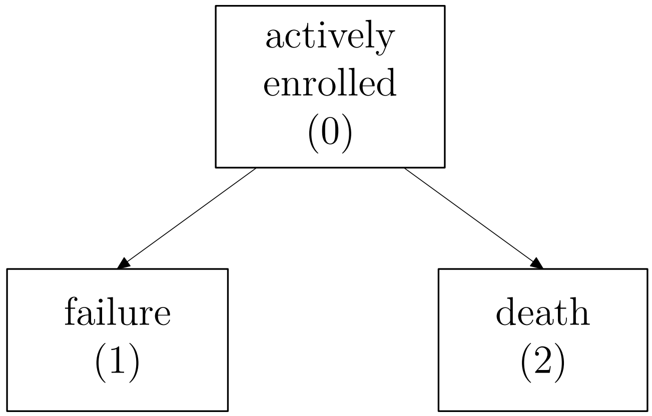
Exercise 8.13

We can actually treat this problem as a double decrement model with state 0, defined to be active while enrolled as a student at the university in good progress. The decrements are state 1 (failure and therefore leaves the university) and state 2 (death). This is best depicted in the figure below.



The APV of the fee income then can be expressed as

$$\text{APV(fee income)} = 10000 \times \sum_{k=0}^{7} (1.02)^k \, v^{k/2} \, {}_{k/2}p_{19}^{00}$$

where

$${}_{k/2}p_{19}^{00} = \mathrm{e}^{-\int_0^{k/2} \left(\mu_{19+t}^{01} + \mu_{19+t}^{02}\right) dt} = {}_{k/2}p_{19}^{(-1)} \times \mathrm{e}^{-\int_0^{k/2} \mu_{19+t}^{02} dt}$$

with ${}_{k/2}p_{19}^{(-1)}$ being our special symbol for continuing to be active and not failing (needed this given the nature of the givens). The calculation details for the probability of continuing to be active can be summarized in a table below:

k	${}_{k/2}p_{19}^{00} = {}_{k/2}p_{19}^{(-1)} \times \mathrm{e}^{-\int_0^{k/2} \mu_{19+t}^{02} dt}$
0	$1.00 \times 1.00 = 1.0000000$
1	$0.85 \times \exp\left[-\left(5 \times 10^{-5} \cdot (19/2)\right)\right] = 0.8495963$
2	$0.85^2 \times \exp\left[-\left(5 \times 10^{-5} \cdot 19\right)\right] = 0.7218140$
3	$(0.85^2)(0.9) \times \exp\left[-\left(5 \times 10^{-5} \cdot (19 + (20/2))\right)\right] = 0.6493078$
4	$(0.85^2)(0.9^2) \times \exp\left[-\left(5 \times 10^{-5} \cdot (19 + 20)\right)\right] = 0.5840849$
5	$(0.85^2)(0.9^2)(0.95) \times \exp\left[-\left(5 \times 10^{-5} \cdot (19 + 20 + (21/2))\right)\right] = 0.5545894$
6	$(0.85^2)(0.9^2)(0.95^2) \times \exp\left[-\left(5 \times 10^{-5} \cdot (19 + 20 + 21)\right)\right] = 0.5265834$
7	$(0.85^2)(0.9^2)(0.95^2)(0.98) \times \exp\left[-\left(5 \times 10^{-5} \cdot (19 + 20 + 21 + (22/2))\right)\right] = 0.5157680$

The result gives us an APV of

$$\text{APV(fee income)} = 53285.18.$$

The R code for the calculations is detailed below:

```
p01 <- c(1,.85,.85,.9,.9,.95,.95,.98)
mu02 <- 5*10^(-5)*(19:22)
p02 <- exp(-0.5*c(0, mu02[1],mu02[1],mu02[2],mu02[2],mu02[3],mu02[3],mu02[4]))
k <- 0:7
i <- 0.05
v <- 1/(1+i)
vk <- v^(k/2)
inck <- 1.02^k
APVfee <- 10000*sum(inck*vk*cumprod(p01)*cumprod(p02))

> cumprod(p01)
[1] 1.0000000 0.8500000 0.7225000 0.6502500 0.5852250 0.5559637 0.5281656 0.5176023
> cumprod(p02)
[1] 1.0000000 0.9995251 0.9990505 0.9985511 0.9980519 0.9975281 0.9970045 0.9964563
> cumprod(p01)*cumprod(p02)
[1] 1.0000000 0.8495963 0.7218140 0.6493078 0.5840849 0.5545894 0.5265834 0.5157680
> APVfee
[1] 53285.18
```