Exercise 8.11

Denote the monthly premium by P so that the APV of future premiums can be expressed as

$$\text{APV}(\text{FP}_0) = P \sum_{k=0}^{239} v^{k/12} \, {}_{k/12}p_{30}^{00}.$$

The APV of future benefits is given by

$$\text{APV}(\text{FB}_0) = 50000 \times \int_0^{20} v^t \, {}_tp_{30}^{00} \, \mu_{30+t}^{01} dt + 75000 \times \int_0^{20} v^t \, {}_tp_{30}^{00} \, \mu_{30+t}^{02} dt.$$

The sums and integrals have to be numerically evaluated with the integrals being approximated using repeated Simpson's rule. Just as in Exercise 8.5, note that

$$\begin{aligned} {}_tp_x^{00} &= \exp\left[-\int_0^t \left(\mu_{x+s}^{01} + \mu_{x+s}^{02}\right) ds\right] \\ &= \exp\left[-1.05\left(\int_0^t \mu_{x+s}^{01} ds\right)\right] \\ &= \exp\left[-1.05\left(At + \frac{Bc^x}{\log(c)}\left(c^t - 1\right)\right)\right] \end{aligned}$$

Solving for the monthly premium, we get

$$P = \frac{50000(0.08287766) + 75000(0.004143883)}{159.0422} = 28.00939.$$

Repeated Simpson's rule with step size $h = 1/1000$ has been used to approximate the integral. The details of the calculations coded in R are given below:

```
x <- 30
A <- 0.0001
B <- 0.00035
c <- 1.075
mux01 <- function(t){
out <- A + B*c^(x+t)
out}
mux02 <- function(t){
out <- 0.05*mux01(t)
out}
tpx00 <- function(t){
temp <- B*c^x*(c^t - 1)/log(c)
out <- exp(-1.05*(A*t + temp))
out}
i <- 0.04
v <- 1/(1+i)
h <- 1/1000
t <- seq(0,20,h)
k <- seq(0,20-(1/12),1/12)
```

```
num1t <- v^t * tpx00(t) * mux01(t)
num2t <- v^t * tpx00(t) * mux02(t)
dentk <- v^k * tpx00(k)
apvfb1 <- 0
apvfb2 <- 0
apvfp <- sum(dentk)
n <- 1
while (n<length(t)) {
n <- n+2
apvfb1 <- apvfb1 + (h/3)*(num1t[n-2]+4*num1t[n-1]+num1t[n])
apvfb2 <- apvfb2 + (h/3)*(num2t[n-2]+4*num2t[n-1]+num2t[n])
}
P <- (50000*apvfb1 + 75000*apvfb2)/apvfp

> apvfb1
[1] 0.08287766
> apvfb2
[1] 0.004143883
> apvfp
[1] 159.0422
> P
[1] 28.00939
```