Exercise 7.6

(a) Let G be the annual gross premium. The actuarial present value of future premiums at issue is

$$\text{APV(FP)} = G\ddot{a}_{[35]:\overline{20}|}.$$

The actuarial present value of future benefits at issue is

$$\text{APV(FB)} = 100000A^{\,1}_{[35]:\overline{20}|}$$

and that of expenses is

$$\text{APV(FE)} = 200 + 0.11G + 0.04G\ddot{a}_{[35]:\overline{20}|}.$$

By the actuarial equivalence principle, we have

$$G\ddot{a}_{[35]:\overline{20}|} = 100000A^{\,1}_{[35]:\overline{20}|} + 200 + 0.11G + 0.04G\ddot{a}_{[35]:\overline{20}|}$$

and solving for G, we have

$$G = \frac{100000A^{\,1}_{[35]:\overline{20}|} + 200}{0.96\ddot{a}_{[35]:\overline{20}|} - 0.11}.$$

Substituting the values

$$\ddot{a}_{[35]:\overline{20}|} = 13.02489$$

and

$$A^{\,1}_{[35]:\overline{20}|} = 0.009324444,$$

we get

$$G = \frac{100000(0.009324444) + 200}{0.96(13.02489) - 0.11} = 91.37115.$$

(b) The policy value immediately following the first premium is

$${}_{0^+}V = G - 200 - 0.15G = 0.85G - 200 = 0.85(91.37115) - 200 = -122.3345.$$

(c) Simply put, the annual gross premium is not sufficient to cover the large initial expenses.

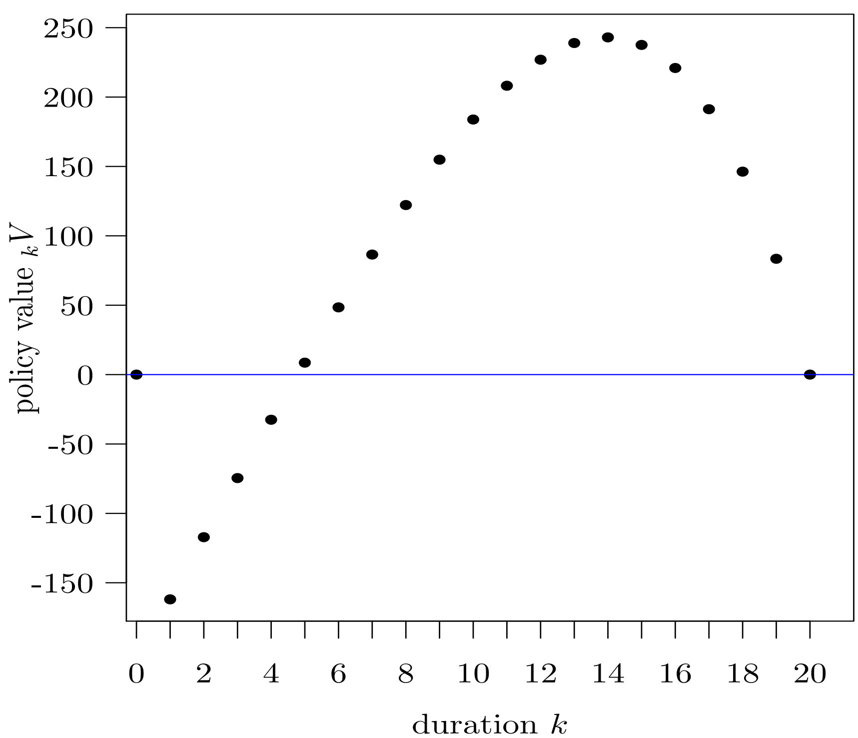
(d) The policy values at each duration just before and just after the premium payment and related expenses are incurred are summarized below:

k	${}_kV$	${}_{k^+}V$	k	${}_kV$	${}_{k^+}V$
0	0.0000	-122.3345			
1	-161.9420	-74.2257	11	208.1524	295.8687
2	-117.1516	-29.4353	12	226.9089	314.6252
3	-74.5746	13.1417	13	238.9530	326.6693
4	-32.5339	55.1824	14	242.9939	330.7102
5	8.6128	96.3291	15	237.5551	325.2714
6	48.4490	136.1653	16	220.9493	308.6656
7	86.4913	174.2076	17	191.2494	278.9657
8	122.1797	209.8960	18	146.2563	233.9726
9	154.8679	242.5842	19	83.4605	171.1768
10	183.8109	271.5273	20	0.0000	

Here we obtained the policy values just after the premium and expenses by adding the applicable premium and expenses incurred at the beginning of each duration. In effect, we have ${}_{0^+}V = {}_0V + 0.85G - 200$, and for $k = 1, 2, \ldots, 19$, we have

$$ {}_{k^+}V = {}_kV + 0.96G. $$

And at expiry, for term insurance, ${}_{20}V = 0$. From the results of the table above, we observe that the policy value first becomes positive at duration 4+. This is also depicted in the figure below.



(e) The following table provides the details of the calculation of the asset shares per surviving policyholder at the beginning of each year. In the calculations, it shows the total for all related cashflows for a portfolio of N policies sold at issue. Each item in the cashflow calculation is multiplied by this number N, but it becomes irrelevant when asset shares are calculated per surviving policyholder because the N cancels in both the numerator and denominator.

Year k	Fund at start of year	Cashflow at start of year	Fund at end of year before death claims	Death claims	Fund at end of year	number of survivors	Asset shares AS_k
1	0.00 N	-122 N	-128 N	33 N	-162 N	0.999666 N	-161.94
2	-161.94 N	-74 N	-78 N	39 N	-117 N	0.999608 N	-117.15
3	-117.15 N	-29 N	-31 N	44 N	-75 N	0.999564 N	-74.57
4	-74.57 N	13 N	14 N	46 N	-33 N	0.999537 N	-32.53
5	-32.53 N	55 N	58 N	49 N	9 N	0.999507 N	8.61
6	8.61 N	96 N	101 N	53 N	48 N	0.999473 N	48.45
7	48.45 N	136 N	143 N	57 N	86 N	0.999435 N	86.49
8	86.49 N	174 N	183 N	61 N	122 N	0.999392 N	122.18
9	122.18 N	210 N	220 N	66 N	155 N	0.999344 N	154.87
10	154.87 N	243 N	255 N	71 N	184 N	0.999290 N	183.81
11	183.81 N	272 N	285 N	77 N	208 N	0.999229 N	208.15
12	208.15 N	296 N	311 N	84 N	227 N	0.999161 N	226.91
13	226.91 N	315 N	330 N	92 N	239 N	0.999084 N	238.95
14	238.95 N	327 N	343 N	100 N	243 N	0.998997 N	242.99
15	242.99 N	331 N	347 N	110 N	237 N	0.998900 N	237.56
16	237.56 N	325 N	342 N	121 N	221 N	0.998791 N	220.95
17	220.95 N	309 N	324 N	133 N	191 N	0.998669 N	191.25
18	191.25 N	279 N	293 N	147 N	146 N	0.998531 N	146.26
19	146.26 N	234 N	246 N	162 N	83 N	0.998377 N	83.46
20	83.46 N	171 N	180 N	180 N	0 N	0.998203 N	0.00

Observe the identical values of the policy values and the asset shares at each duration. Thus, it is clear from this example that if the actual experience follows that of the assumption basis used in the premium/policy value calculations, the two should be identical.