Exercise 7.3

(a) Let G be the annual gross premium. The APV of future gross premiums is therefore

$$\text{APV(FP)} = G\ddot{a}_{[35]}$$

and the APV of future benefits and expenses is

$$\text{APV(FB)} + \text{APV(FE)} = 100000A_{[35]} + 0.35G + 0.05G\ddot{a}_{[35]} + 85 + 40\ddot{a}_{[35]}.$$

Equating the two APV's we get

$$G = \frac{100000A_{[35]} + 40\ddot{a}_{[35]} + 85}{0.95\ddot{a}_{[35]} - 0.35}$$

where (one can easily verify with an R code or an excel)

$$\ddot{a}_{[35]} = 16.52456$$

and

$$A_{[35]} = 1 - (1 - 1.06^{-1}) \times 16.52456 = 0.0646478.$$

Substituting these values, we have

$$G = \frac{100000(0.0646478) + 40(16.52456) + 85}{0.95(16.52456) - 0.35} = 469.8077.$$

(b) The net annual premium is given by

$$P = 100000 \times \frac{A_{[35]}}{\ddot{a}_{[35]}} = 100000 \times \frac{0.0646478}{16.52456} = 391.2227.$$

With a starting value of ${}_0V^n = 0$, we use the recursive formula to compute the following year's net premium policy value:

$$({}_0V^n + P)(1+i) = {}_1V^n + q_{[35]}(100000 - {}_1V^n)$$

With $q_{35]} = 0.0003343660$, this leads us to

$${}_1V^n = \frac{(0 + 391.2227)(1.06) - 100000(0.0003343660)}{1 - 0.0003343660} = 381.3869.$$

(c) With a starting value of ${}_0V^g = 0$ (because G has been determined by the equivalence principle), we follow a similar recursive formula to compute the following year's gross premium policy value:

$${}_1V^g = \frac{[0 + 0.6(469.8077) - 125](1.06) - 100000(0.0003343660)}{1 - 0.0003343660} = 132.9055.$$

(d) Because expenses are not level and with rather larger first year than renewal expenses, the future gross premiums allow you to recover these first year expenses by amotizing them throughout the premium paying period. This explains why the gross premium policy value is smaller than the net premium policy value, even using the same mortality and interest basis.

(e) With interest of 5.5% per year, one can verify

$$\ddot{a}_{[35]} = 17.67178$$

and

$$A_{[35]} = 1 - (1 - 1.055^{-1}) \times 17.67178 = 0.07872234$$

so that

$$G = \frac{100000(0.07872234) + 40(17.67178) + 85}{0.95(17.67178) - 0.35} = 527.0717$$

and the corresponding gross premium policy value is

$${}_1V^g = \frac{[0 + 0.6(527.0717) - 125](1.055) - 100000(0.0003343660)}{1 - 0.0003343660} = 169.3376.$$

(f) With a starting asset share value of $\mathrm{AS}_0 = 0$, we use the recursion formula to arrive at the next year's asset share:

$$\mathrm{AS}_1 = \frac{[0 + 0.6(469.8077) - 125](1.06) - 100000(0.0003343660)}{1 - 0.0003343660} = 132.9055.$$

Because experience is assumed to follow exactly that of the premium basis, this results in the same value as the gross premium policy value.

(g) Using the given experience, the next year's asset share is

$$\mathrm{AS}_1 = \frac{[0 + 0.6(469.8077) - 125 - 25](1.10) - 100000(0.0012)}{1 - 0.0012} = 25.103225.$$

Although the interest earned is actually larger than that of the premium basis, this leads to a much smaller asset share because expenses and mortality are much worse.

(h) The insurer's accumulated assets at the end of the year is

$$[0 + 0.6(469.8077) - 150] \times 1.10 = 145.0731.$$

From (c), we know that ${}_1V^g = 132.9055$. The actual death claims plus the policy value held for surviving policies is given by

$$100000(0.0012) + 132.9055(1 - 0.0012) = 252.7461.$$

The insurer's profit per policy is therefore the difference between these two:

$$145.0731 - 252.7461 = -107.673.$$

(i) The profit in (h) can be broken down due to:

1. gain from interest

$$[0 + 0.6(469.8077) - 125] \times (0.10 - 0.06) = 6.275385$$

2. loss from higher actual than expected expenses

$$(125 - 150) \times 1.10 = -27.5$$

3. loss from mortality

$$(0.0003343660 - 0.0012) \times (100000 - 132.9055) = -86.44835$$

The sum of these three components lead us to the surplus calculated in (h). Note that we have a loss because the losses due to worse expenses and worse mortality outweighs the gain from higher than expected investment earnings.