Exercise 7.20

(a) Calculating the first year FPT premium, we have

$$ {}_1P_{[50]} = 100,000 \times vq_{[50]} = 100,000 \times (1/1.04) \times [1-(98450.67/98552.51)] = 99.36132 $$

If we let β to be the renewal premium, then it is clear that

$$ \beta = \frac{P\ddot{a}_{[50]:\overline{20}|} - {}_1P_{[50]}}{\ddot{a}_{[50]:\overline{20}|} - 1} $$

where P is the net annual premium equal to

$$ P = 100,000\frac{A_{[50]}}{\ddot{a}_{[50]:\overline{20}|}} = 100,000\frac{0.255698}{13.86135} = 1844.684. $$

Plugging the appropriate values, we get

$$ \beta = \frac{1844.684(13.86135) - 99.36132}{13.86135 - 1} = 1980.386. $$

(b) First, consider gross premium valuation. At issue, the APV of future gross premiums is

$$ \text{APV(FG)} = G\ddot{a}_{[50]:\overline{20}|} $$

and the APV of future benefits is

$$ \text{APV(FB)} = 100000 \times A_{[50]} $$

and the APV of future expenses is

$$ \text{APV(FE)} = 0.47G + 225 + 0.03G\ddot{a}_{[50]:\overline{20}|} + 25\ddot{a}_{[50]:\overline{20}|} $$

Thus, from equivalence principle, we have

$$ G = \frac{100000 \times A_{[50]} + 225 + 25\ddot{a}_{[50]:\overline{20}|}}{0.9725\ddot{a}_{[50]:\overline{20}|} - 0.47} = \frac{26141.33}{12.97551} = 2014.668. $$

Thus, the gross premium reserves for $t = 0, 1, 2$ and 10 are:

$$ {}_0V^g = 0 $$

$$ {}_1V^g = \frac{({}_0V^g + 0.5G - 250)(1.04) - 100000q_{[50]}}{1 - q_{[50]}} = 684.9992 $$

where $q_{[50]} = 0.001033358$

$$ {}_2V^g = \frac{({}_1V^g + 0.97G - 25)(1.04) - 100000q_{[50]+1}}{1 - q_{[50]+1}} = 2595.639 $$

where $q_{[50]+1} = 0.00126439$

Finally for $t = 10$, we have

$$\begin{aligned} {}_{10}V^g &= 100000A_{60} + (25 - 0.97G)\ddot{a}_{60:\overline{10}|} \\ &= 100000(0.3629975) + (25 - 0.97(2014.668))(8.273434) \\ &= 20338.41 \end{aligned}$$

For net premium valuation, we have the net annual premium equal to $P = 1844.684$ from part (a). Thus, the net premium reserves for $t = 0, 1, 2$ and 10 are:

$$ {}_0V^n = 0 $$

$$ {}_1V^n = \frac{(0 + P)(1.04) - 100000q_{[50]}}{1 - q_{[50]}} = 1817.013 $$

$$ {}_2V^n = \frac{({}_1V^n + P)(1.04) - 100000q_{[50]+1}}{1 - q_{[50]+1}} = 3686.386 $$

$$ {}_{10}V^n = 100000A_{60} - P\ddot{a}_{60:\overline{10}|} = 100000(0.3629975) - (1844.684)(8.273434) = 21037.88 $$

For FPT reserve calculation, we need the first and renewal year's premiums computed in (a):

$$ \alpha = 99.36132 \qquad \beta = 1980.386 $$

Thus, the FPT reserves for $t = 0, 1, 2$ and 10 are:

$$ {}_0V^{\text{FPT}} = 0 $$

$$ {}_1V^{\text{FPT}} = 0 $$

$$ {}_2V^{\text{FPT}} = \frac{({}_1V^{\text{FPT}} + \beta)(1.04) - 100000q_{[50]+1}}{1 - q_{[50]+1}} = 1935.61 $$

$$ {}_{10}V^{\text{FPT}} = 100000A_{60} - \beta\ddot{a}_{60:\overline{10}|} = 100000(0.3629975) - (1980.386)(8.273434) = 19915.15 $$