

Exercise 7.19

- (a) Let G be the annual gross premium for the first 5 years. The APV of future gross premiums is therefore

$$\text{APV}(\text{FG}) = G (\ddot{a}_{[40]} - 0.5_5E_{[40]}\ddot{a}_{45} - 0.5_{20}E_{[40]}\ddot{a}_{60})$$

and the APV of future benefits is

$$\text{APV}(\text{FB}) = 600000 (A_{[40]} - 0.5_5E_{[40]}A_{45} - 0.5_{20}E_{[40]}A_{60})$$

and the APV of future expenses is

$$\text{APV}(\text{FE}) = 200 + 0.40G + 0.10G (\ddot{a}_{[35]} - 0.5_5E_{[40]}\ddot{a}_{45} - 0.5_{20}E_{[40]}\ddot{a}_{60})$$

Using the equivalence principle and solving for G :

$$G = \frac{600000 (A_{[35]} - 0.5_5E_{[40]}A_{45} - 0.5_{20}E_{[40]}A_{60}) + 200}{0.9 (\ddot{a}_{[35]} - 0.5_5E_{[40]}\ddot{a}_{45} - 0.5_{20}E_{[40]}\ddot{a}_{60}) - 0.4}$$

Plug the following values (taken directly from the Standard Select/Ultimate Tables on pp. 584-587:

$$A_{[40]} = 0.12097 \quad A_{45} = 0.15161 \quad A_{60} = 0.29028$$

$$\ddot{a}_{[40]} = 18.460 \quad \ddot{a}_{45} = 17.816 \quad \ddot{a}_{60} = 14.904$$

$${}_5E_{[40]} = 0.78121 \quad {}_{20}E_{[40]} = 0.36667$$

We get

$$G = \frac{5319.135}{7.491701} = 710.0037$$

- (b) Because the valuation interest rate is 4.5%, instead of 5%, the gross premium reserve is not equal to 0 at issue. Instead, we re-calculate the APV's at this new interest rate. At issue, the APV of future gross premiums is therefore

$$\text{APV}(\text{FG}) = G (\ddot{a}_{[40]} - 0.5_5E_{[40]}\ddot{a}_{45} - 0.5_{20}E_{[40]}\ddot{a}_{60}) = 6418.329$$

and the APV of future benefits is

$$\text{APV}(\text{FB}) = 600000 (A_{[40]} - 0.5_5E_{[40]}A_{45} - 0.5_{20}E_{[40]}A_{60}) = 5391.965$$

and the APV of future expenses is

$$\text{APV}(\text{FE}) = 200 + 0.40G + 0.10G (\ddot{a}_{[35]} - 0.5_5E_{[40]}\ddot{a}_{45} - 0.5_{20}E_{[40]}\ddot{a}_{60}) = 1125.834$$

These values were used and were calculated based on interest rate of 4.5%:

$$A_{[40]} = 0.1461531 \quad A_{45} = 0.1794577 \quad A_{60} = 0.3241292$$

$$\ddot{a}_{[40]} = 19.82822 \quad \ddot{a}_{45} = 19.05482 \quad \ddot{a}_{60} = 15.69522$$

$${}_5E_{[40]} = 0.8000768 \quad {}_{20}E_{[40]} = 0.4033981$$

Thus, the gross premium reserves for $t = 0, 1, 2$ are:

$${}_0V^g = 5391.965 + 1125.834 - 6418.329 = 99.4709$$

$${}_1V^g = \frac{({}_0V^g + 0.5G - 200)(1.045) - 600000q_{[40]}}{1 - q_{[40]}} = -4.455405$$

where $q_{[40]} = 0.000450629$

$${}_2V^g = \frac{({}_1V^g + 0.9G)(1.045) - 600000q_{[40]+1}}{1 - q_{[40]+1}} = 341.1764$$

where $q_{[40]+1} = 0.0005368489$

For net premium reserve calculation, we first re-calculate the net premium at 4.5%:

$$P = \frac{600000 \left(A_{[40]} - 0.5{}_5E_{[40]}A_{45} - 0.5{}_{20}E_{[40]}A_{60} \right)}{\ddot{a}_{[40]} - 0.5{}_5E_{[40]}\ddot{a}_{45} - 0.5{}_{20}E_{[40]}\ddot{a}_{60}} = \frac{5391.965}{9.039852} = 596.4661$$

Thus, the net premium reserves for $t = 0, 1, 2$ are:

$${}_0V^n = 0$$

$${}_1V^n = \frac{(0 + P)(1.045) - 600000q_{[40]}}{1 - q_{[40]}} = 353.0887$$

where $q_{[40]} = 0.000450629$

$${}_2V^n = \frac{({}_1V^n + P)(1.045) - 600000q_{[40]+1}}{1 - q_{[40]+1}} = 670.5354$$

where $q_{[40]+1} = 0.0005368489$.

For FPT reserve calculation, we need to compute the first and renewal year's premiums:

$$\alpha = 600000vq_{[40]} = 258.7344$$

$$\beta = \frac{P \left(\ddot{a}_{[35]} - 0.5{}_5E_{[40]}\ddot{a}_{45} - 0.5{}_{20}E_{[40]}\ddot{a}_{60} \right) - \alpha}{\left(\ddot{a}_{[35]} - 0.5{}_5E_{[40]}\ddot{a}_{45} - 0.5{}_{20}E_{[40]}\ddot{a}_{60} \right) - 1} = 638.4733$$

Thus, the FPT reserves for $t = 0, 1, 2$ are:

$${}_0V^{\text{FPT}} = 0$$

$${}_1V^{\text{FPT}} = 0$$

$${}_2V^{\text{FPT}} = \frac{({}_1V^{\text{FPT}} + \beta)(1.045) - 600000q_{[40]+1}}{1 - q_{[40]+1}} = 345.2806$$

where $q_{[40]+1} = 0.0005368489$

- (c) The FPT reserve valuation is a modified reserve system to provide for allowance of expenses for which the net premium valuation does not. The FPT has been too simplified by allowing for a small first year premium and a larger renewal premiums to help achieve the goal of providing for expenses. In essence, the first year premium is calculated to just have enough to cover for the first year cost of insurance. In order for these modified reserve valuation methods to be effective, the modified stream of reserves should approximately be close to the gross premium reserves. In this case, this does not seem to be achieved in the first two years.