Exercise 7.16

(a) Let P be the annual benefit premium, per contract, payable during the 10-year deferred period. The actuarial present value of future premiums at issue is

$$\text{APV}(\text{FP}_0) = P\ddot{a}_{[60]:\overline{10}|}.$$

The actuarial present value of future benefits at issue, which includes the death benefit during the deferred period and the deferred annuity after 10 years, is

$$\text{APV}(\text{FB}_0) = 50000A^{\;1}_{[60]:\overline{10}|} + 10000\,{}_{10}E_{[60]}\,\ddot{a}_{70}.$$

By the actuarial equivalence principle, we have

$$\begin{aligned} P &= 10000 \times \frac{5A^{\;1}_{[60]:\overline{10}|} + {}_{10}E_{[60]}\,\ddot{a}_{70}}{\ddot{a}_{[60]:\overline{10}|}} \\ &= 10000 \times \frac{5(0.03958813) + 0.5266743(11.13150)}{7.662697} \\ &= 7909.249. \end{aligned}$$

The actuarial values above are based on the Standard Select Survival Model with $i = 6\%$.

(b) Starting with ${}_0V = 0$, applying recursion relation, the policy values can be expressed as:

$${}_kV = \frac{({}_{k-1}V + P)(1.06) - 50000q_{[60]+k-1}}{1 - q_{[60]+k-1}}, \quad \text{for } k = 1, 2, \ldots, 9$$

$${}_{10}V = \frac{({}_9V + P)(1.06) - (50000q_{69} + 10000p_{69})}{1 - q_{69}},$$

and

$${}_kV = \frac{(1.06)\,{}_{k-1}V - 10000(1 - q_{[60]+k-1})}{1 - q_{[60]+k-1}}, \quad \text{for } k = 11, 12, \ldots$$

(c) To calculate the death strain in year 3, we need the policy values at the end of year 3. Applying the recursion formulas above, we have:

$${}_1V = \frac{(7909.249)(1.06) - 50000(0.002906289)}{1 - 0.002906289} = 8262.502$$

$${}_2V = \frac{(8262.502 + 7909.249)(1.06) - 50000(0.003602519)}{1 - 0.003602519} = 17023.260$$

and

$${}_3V = \frac{(17023.260 + 7909.249)(1.06) - 50000(0.0042336)}{1 - 0.0042336} = 26328.240$$

Therefore, the death strain at risk in year 3 is

$$50000 - {}_3V = 50000 - 26328.24 = 23671.76.$$

(d) One can also verify that at the end of year 10, the policy value is

$$_{10}V = 10000 \times \ddot{a}_{70} = 10000(11.13150) = 111315.0.$$

This is because there are only the annuity benefits to be paid in the future starting at age 70 and no more premiums to be paid. Similarly applies for subsequent policy values:

$$_{11}V = 10000 \times \ddot{a}_{71} = 10000(10.85240) = 108524.0$$

$$_{12}V = 10000 \times \ddot{a}_{72} = 10000(10.56686) = 105668.6$$

$$_{13}V = 10000 \times \ddot{a}_{73} = 10000(10.27528) = 102752.8$$

Therefore, the death strain at risk in year 13 is

$$0 - {}_{13}V = 0 - 102752.8 = -102752.8.$$

(e) At the end of year 3, the actual policy value for the portfolio is given by

$$97 \times ({}_{2}V + P)(1.06) - 3 \times 50000 = 2413560$$

and the expected policy value for the portfolio is given by

$$(97 - 3) \times {}_{3}V = 2474854$$

The difference between the expected and actual policy values gives the mortality profit for the year:

$$2474854 - 2413560 = 61294.26.$$

This is a loss because we expected fewer deaths than actual. Indeed, our expected number of deaths in year 3 can be found using

$$97 \times q_{62} = 97 \times 0.0042336 = 0.4106592$$

(e) At the end of year 13, the actual policy value for the portfolio is given by

$$(80 - 4) \times {}_{13}V = 76 \times 102752.8 = 7809215$$

and the expected policy value for the portfolio is given by

$$80 \times (1 - q_{72}) \times {}_{13}V = 80 \times (1 - 0.01308068) \times 102752.8 = 8112701$$

The difference between the expected and actual policy values gives the mortality profit for the year:

$$8112701 - 7809215 = 303485.2.$$

This is a gain because we expected more survivors than actual. Indeed, our expected number of survivors at the end of year 13 can be found using

$$80 \times (1 - q_{72}) = 80 \times (1 - 0.01308068) = 78.95355.$$

We expected 79 survivors by the end of year 13; 76 actually survived.