Exercise 7.13

(a) Let P be the annual benefit premium. The actuarial present value of future premiums at issue is

$$\text{APV}(\text{FP}_0) = P\ddot{a}_{[40]}.$$

The actuarial present value of future benefits at issue is

$$\text{APV}(\text{FB}_0) = 1000A_{[40]} + 49000\,{}_{10}E_{[40]}A_{43}.$$

Using the actuarial equivalence principle and substituting values, we have

$$\begin{aligned} P &= \frac{1000A_{[40]} + 49000\,{}_{10}E_{[40]}A_{43}}{\ddot{a}_{[40]}} \\ &= \frac{1000(0.08424182) + 49000(0.8382802)(0.09880822)}{16.17839} = 256.0737. \end{aligned}$$

(b) For $t \geq 3$, the policy value can be expressed as

$${}_tV = 50000A_{[40]+t} - P\ddot{a}_{[40]+t}.$$

(c) The policy value at $t = 3$ is

$${}_3V = 50000A_{43} - P\ddot{a}_{43} = 50000(0.09880822) - 256.0737(15.92105) = 863.4472.$$

(d) Starting with ${}_0V = 0$, applying recursion relation, we have the policy values for the first two year as:

$$\begin{aligned} {}_1V &= \frac{({}_0V + P)(1+i) - 1000q_{[40]}}{1 - q_{[40]}} \\ &= \frac{256.0737(1.06) - 1000(0.0004506435)}{1 - 0.0004506435} = 271.1097 \end{aligned}$$

and

$$\begin{aligned} {}_2V &= \frac{({}_1V + P)(1+i) - 1000q_{[40]+1}}{1 - q_{[40]+1}} \\ &= \frac{(271.1097 + 256.0737)(1.06) - 1000(0.0005368943)}{1 - 0.0005368943} = 558.5774. \end{aligned}$$

(e) The expected asset share per surviving policyholder at the end of year 3 is

$$\begin{aligned} \text{EA}_3 &= \frac{({}_2V + P)(1+0.06) - 1000q_{42}}{1 - q_{42}} \\ &= \frac{256.0737(1.06) - 1000(0.0004506435)}{1 - 0.0004506435} = 863.4472 \end{aligned}$$

while the actual asset share per surviving policyholder is

$$\begin{aligned} \mathrm{AA}_3 &= \frac{({}_2V + P)(1 + 0.055) - 1000(4/985)}{1 - (4/985)} \\ &= \frac{256.0737(1.06) - 1000(4/985)}{1 - (4/985)} = 858.8839. \end{aligned}$$

Thus, the total profit for the year is the difference between the two multiplied by the number of actual surviving policyholders:

$$981 \times (858.8839 - 863.4472) = -4476.573.$$