Math 3631 Review Problems 1
27 January 2020

1. For a fully discrete whole life insurance policy of 1 issued to (x), prove the following formulas for net premium reserves:

   - ${}_kV = 1 - \dfrac{\ddot{a}_{x+k}}{\ddot{a}_x}$

   - ${}_kV = 1 - \dfrac{P_x + d}{P_{x+k} + d} = \dfrac{P_{x+k} - P_x}{P_{x+k} + d}$

   - ${}_kV = 1 - \dfrac{1 - A_{x+k}}{1 - A_x} = \dfrac{A_{x+k} - A_x}{1 - A_x}$

2. Assume mortality follows the `Standard Ultimate Life Table` with $i = 0.05$. Calculate the net premium reserves at the end of year 10 for each of the following policies:

   - A fully discrete whole life insurance policy issued to (45) with death benefit equal to 10,000.

   - A 20-year term insurance policy issued to (50) with death benefit of 25,000 payable at the end of the year of death and premiums are payable once at the beginning of each year.

3. For a special whole life insurance on (45), you are given:

   - Benefit is paid at the end of the year of death. The death benefit is \$100,000 for the first 20 years and reduces to \$50,000 thereafter.
   - The annual benefit premium of \$4,945 is payable once at the beginning of each year for the first 20 years only; no premiums are payable after 20 years.
   - The following actuarial present values:

x	A_x	$\ddot{a}_x$	${}_{10}E_x$
55	0.5628	4.8091	0.0758
65	0.7532	2.7147	0.0015

   Calculate the benefit reserve at the end of 10 years.

4. Consider a fully discrete whole life insurance of B to (x). Prove the following recursive formula for reserve calculations:

$$ {}_{k+1}V = \frac{\left({}_kV + P\right)(1+i) - Bq_{x+k}}{1 - q_{x+k}}, $$

   with $k = 1, 2, \ldots$ and ${}_0V = 0$.

5. For a fully discrete whole life insurance of 1,000 on (65), you are given:

   - The net premium reserve at the end of policy year 24 is 502.58.
   - The net premium reserve at the end of policy year 25 is 527.85.
   - $A_{65} = 0.6135$
   - $i = 0.05$

   Calculate q_{89}.

Suggested Solution

① fully discrete means discrete premium (b.o.y.)
discrete benefit (e.o.y.)

$$ {}_kV = APV(FB_k) - APV(FP_k) $$

$$ = A_{x+k} - P_x \cdot \ddot{a}_{x+k} $$

where $P_x = A_x / \ddot{a}_x$

(i) $$ {}_kV = A_{x+k} - \frac{A_x}{\ddot{a}_x}\ddot{a}_{x+k} $$

$$ = (1 - d\ddot{a}_{x+k}) - \frac{1 - d\ddot{a}_x}{\ddot{a}_x}\ddot{a}_{x+k} $$

$$ = 1 - \cancel{d\ddot{a}_{x+k}} - \frac{\ddot{a}_{x+k}}{\ddot{a}_x} + \cancel{d\ddot{a}_{x+k}} = 1 - d\ddot{a}_{x+k} $$

$$ \ddot{a}_x = \frac{1 - A_x}{d} $$

(ii) $$ \cancel{{}_kV = A_{x+k} - \frac{A_x}{\ddot{a}_x}\ddot{a}_{x+k}} $$

$$ \cancel{= A_{x+k} - A_x \cdot \frac{(1 - A_{x+k})/d}{(1 - A_x)/d}} = $$

$$ P_x = A_x / \ddot{a}_x = \frac{1 - d\ddot{a}_x}{\ddot{a}_x} = \frac{1}{\ddot{a}_x} - d $$

$$ {}_kV = 1 - \frac{\ddot{a}_{x+k}}{\ddot{a}_x} $$

$$ \Rightarrow \ddot{a}_x = \frac{1}{P_x + d} $$

$$ = 1 - \frac{1/(P_{x+k} + d)}{1/(P_x + d)} = 1 - \frac{P_x + d}{P_{x+k} + d} \quad \text{OR} \quad \frac{P_{x+k} - P_x}{P_{x+k} + d} $$

(iii) $_kV = 1 - \dfrac{\ddot{a}_{x+k}}{\ddot{a}_x}$ $\qquad \ddot{a}_x = \dfrac{1-A_x}{d}$

$$= 1 - \frac{(1-A_{x+k})/d}{(1-A_x)/d} = 1 - \frac{1-A_{x+k}}{1-A_x}$$

$$= \frac{A_{x+k} - A_x}{1-A_x}$$

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$$P = \frac{10{,}000 A_{45}}{\ddot{a}_{45}} \quad \text{Then} \quad {}_{10}V = 10000 A_{55} - P\,\ddot{a}_{55}$$

(i)

plug values A_{45}, $\ddot{a}_{45}$, A_{55}, $\ddot{a}_{55}$

OR

$${}_{10}V = 10{,}000\left(1 - \frac{\ddot{a}_{55}}{\ddot{a}_{45}}\right) = 10{,}000\left(1 - \frac{16.0599}{17.8162}\right)$$

$$= 985.7882$$

(ii) 20-year term

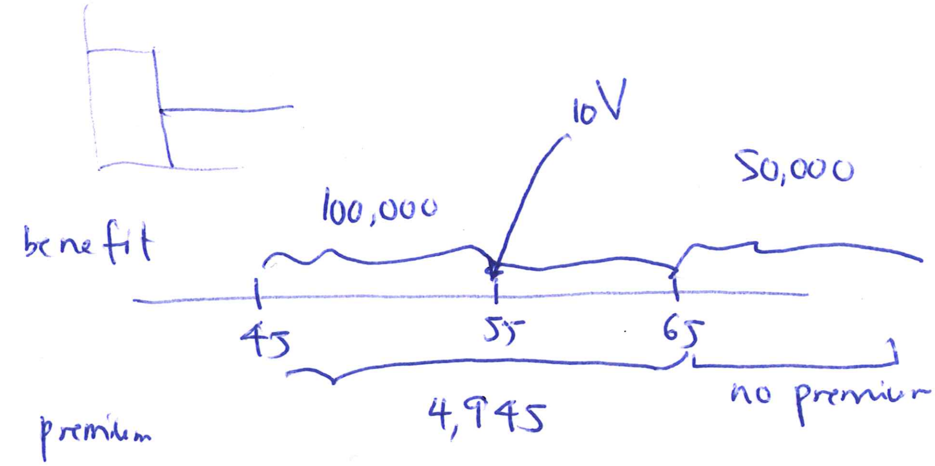
$$P = \frac{\cancel{10{,}000}\ 25{,}000\, A^{1}_{50:\overline{20}|}}{\ddot{a}_{50:\overline{20}|}} = \underbrace{25000\, \frac{\overset{\cancel{.61643}\ .38844}{A_{50:\overline{20}|}} - \overset{.34824}{{}_{20}E_{50}}}{\underset{12.8428}{\ddot{a}_{50:\overline{20}|}}}}_{78.25396}$$

$${}_{10}V = 25000 A^{1}_{60:\overline{10}|} - P\,\ddot{a}_{60:\overline{10}|}$$

$$= 25000(\underset{.62116}{A_{60:\overline{10}|}} - \underset{.57864}{{}_{10}E_{60}}) - P\,\underset{7.9555}{\ddot{a}_{60:\overline{10}|}}$$

$$= \underline{440.4506}$$

③



$$_{10}V = APV(FB_{10}) - APV(FP_{10})$$

$$= \underbrace{100{,}000\left(\underbrace{A_{55}}_{.5628} - 0.5\ \underbrace{{}_{10}E_{55}}_{.0758}\ \underbrace{A_{65}}_{.0015}\right)}_{53425.37} - \underbrace{4945\left(\underbrace{\ddot{a}_{55}}_{4.8091} - {}_{10}E_{55}\ \underbrace{\ddot{a}_{65}}_{2.7197}\right)}_{22{,}763.45}$$

$$= 53425.37$$

$$= 30{,}661.93$$

④ P is determined by $P = B A_x / \ddot{a}_x$

Start with

$${}_kV = B A_{x+k} - P \ddot{a}_{x+k}$$

recall for recursive equation for A_{x+k} , $\ddot{a}_{x+k}$

$$A_{x+k} = v q_{x+k} + v p_{x+k} A_{x+k+1}$$

$$\ddot{a}_{x+k} = 1 + v p_{x+k} \ddot{a}_{x+k+1}$$

$$= B(v q_{x+k} + v p_{x+k} A_{x+k+1}) - P[1 + v p_{x+k} \ddot{a}_{x+k+1}]$$

$$= (B \cdot v q_{x+k} - P) + v p_{x+k} (\underbrace{B A_{x+k+1} - P \ddot{a}_{x+k+1}}_{{}_{k+1}V})$$

Solve for ${}_{k+1}V$:

$${}_{k+1}V = \frac{{}_kV + P - B \cdot v q_{x+k}}{v p_{x+k}}$$

$$= \frac{({}_kV + P)(1+i) - B \cdot q_{x+k}}{1 - q_{x+k}}$$

since $p_{x+k} = 1 - q_{x+k}$

(5) Apply recursive formula from previous problem

$$P = 1000\frac{A_x}{\ddot{a}_x} = 1000\frac{A_x}{\frac{1-A_x}{d}} = \underbrace{1000\cdot\frac{d\,A_x}{1-A_x}}_{P = 75.58677}$$

$A_x = .6135$

$d = \frac{.05}{1.05}$

$$_{25}V = \frac{(_{24}V + P)(1+i) - 1000\,q_{89}}{1-q_{89}}$$

or equivalently

$$_{25}V = (_{24}V + P)(1+i) - (1000 - {_{25}V})\,q_{89}$$

Solve for q_{89}, we have

$$q_{89} = \frac{(_{24}V+P)(1+i) - {_{25}V}}{1000 - {_{25}V}}$$

$$= \frac{(502.58 + 75.58677)(1.05) - 527.85}{1000 - 527.85}$$

$$= \frac{104.2251}{472.15} = \underline{0.2207458}$$