STT 456 Review Problems for Class Test 1
February 25, 2015

1. An insurance company sells 1,000 fully discrete whole life insurance contracts of \$1, each to the same age 50. You are given:

   - All contracts have independent future lifetimes.
   - There are no expenses.
   - Mortality follows the `Standard Ultimate Survival Model` with $i = 5\%$.

   Using the Normal approximation, calculate the annual contract premium, for each policy, according to the portfolio percentile premium principle so that the company has at least a 95% probability of a positive gain from this portfolio of contracts.

2. For a special whole life insurance on (45), you are given:

   - Benefit is paid at the end of the year of death. The death benefit is \$100,000 for the first 20 years and reduces to \$50,000 thereafter.
   - The annual benefit premium of \$4,945 is payable once at the beginning of each year for the first 20 years only; no premiums are payable after 20 years.
   - The following actuarial present values:

x	A_x	$\ddot{a}_x$	${}_{10}E_x$
55	0.5628	4.8091	0.0758
65	0.7532	2.7147	0.0015

   Calculate the benefit reserve at the end of 10 years.

3. For a fully discrete whole life insurance of \$1,000 on (x), you are given:

   - The expense, incurred at the beginning of each year, is 10% of the annual benefit premium.
   - The gross premium reserve at the end of policy year k is 602.45.
   - The gross premium reserve at the end of policy year $k+1$ is 629.72.
   - $A_x = 0.6135$
   - $i = 5\%$

   Calculate q_{x+k}.

4. An insurer issued 400,000 fully discrete whole life insurance policies to lives all exactly age 50 on January 1, 2002. Each policy issued has a death benefit of \$100,000 with an annual gross premium of \$2,600.

   You are given:

- The following values in Year 2011:

	anticipated	actual
Expenses as a percent of premium	0.05	0.06
Annual effective rate of interest	0.02	0.05
q_{59}	0.0085	0.0090

- The gross premium reserves per policy at the end of Year 2010 and Year 2011, respectively, are:

$$_{9}V = 2,044.32 \text{ and } _{10}V = 2,324.13$$

- A total of 385,100 remain in force at the beginning of Year 2011.
- Gains and losses are calculated in the following order: interest then expenses then mortality.

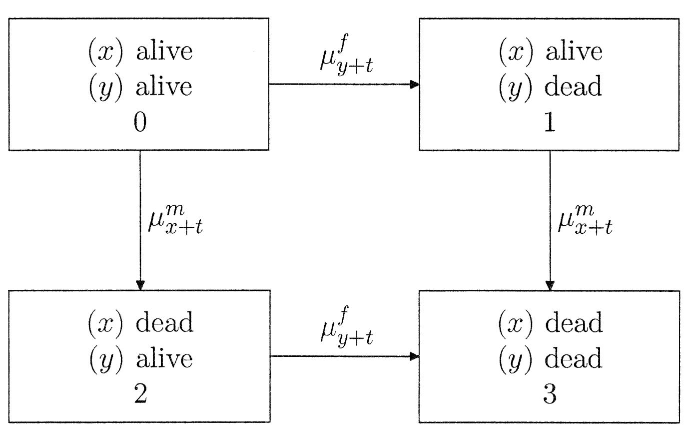
Calculate the total gain (or loss) due to interest for this portfolio of policies in Year 2011.

5. For a life insurance policy issued to (x), you are given:

   - Death benefit of \$1 is paid at the end of the year of death.
   - The benefit premium in year 11, payable at the beginning of the year, is \$0.045.
   - There are no expenses for this policy.
   - The policy is still active after 10 years.
   - Deaths are assumed to be uniformly distributed over integral ages.
   - $_{10}V = 0.325$
   - $p_{x+10} = 0.925$
   - $i = 6\%$

   Calculate $_{10.4}V$.

6. The joint lifetime of a husband (x) and a wife (y) is being modeled as:



You are given:

$$\mu^{m}_{x+t} = 0.03, \text{ for all } t > 0$$

and

$$\mu^{f}_{y+t} = 0.02, \text{ for all } t > 0$$

Calculate the probability that (x) and (y), given both are alive today, will be dead within the next 10 years.

Review Session 2/25/15

1

Question No. 3:

An insurance company sells 1,000 fully discrete whole life insurance contracts of \$1, each to the same age 50. You are given:

- All contracts have independent future lifetimes.
- There are no expenses.
- Mortality follows the `Standard Ultimate Survival Model` with $i = 5\%$.

give values

Using the Normal approximation, calculate the annual contract premium, for each policy, according to the portfolio percentile premium principle so that the company has at least a 95% probability of a positive gain from this portfolio of contracts.

Let P be the required premium for each policy K

$$L_{0,K} = PVFB_0 - PVFP_0 = v^{K+1} - P\ddot{a}_{\overline{K+1|}}$$

the K here denotes the contract lifetime

$$E[L_{0,K}] = A_{50} - P\ddot{a}_{50} = 0.18931 - P(17.0245)$$

$$Var[L_{0,K}] = \left(1 + P/d\right)^2 \left[{}^2A_{50} - (A_{50})^2\right]$$

$$= \left(1 + \frac{P}{.05/1.05}\right)^2 \underbrace{\left[.05108 - (0.18931)^2\right]}_{0.01524172}$$

$$= (1 + 21P)^2 (0.01524172)$$

Let $L_{agg} = \sum_{K=1}^{1000} L_{0,K}$

$$E[L_{agg}] = 1000\, E[L_{0,K}] = 1000(.18931 - P(17.0245))$$
$$= 189.31 - 17024.5P$$

$$Var[L_{agg}] = 1000\, Var[L_{0,K}] = 1000(1+21P)^2(0.01524172)$$
$$= 15.24172(1+21P)^2$$

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$$\Pr[L_{agg} < 0] \geq 0.95 \iff \Pr\left[Z < \underbrace{\frac{-(189.31 - 17024.5P)}{\sqrt{15.24172\,(1+21P)^2}}}_{1.645}\right] \geq 0.95$$

Solving for P, we get

$$-189.31 + 17024.5P \geq \underbrace{1.645\sqrt{15.24172}}_{6.422187}(1+21P)$$

$$(\underbrace{17024.5 - 6.422187(21)}_{16889.63})P \geq \underbrace{6.422187 + 189.31}_{195.7322}$$

$$P \geq \frac{195.7322}{16889.63} = \underline{\underline{0.01158889}}$$

2

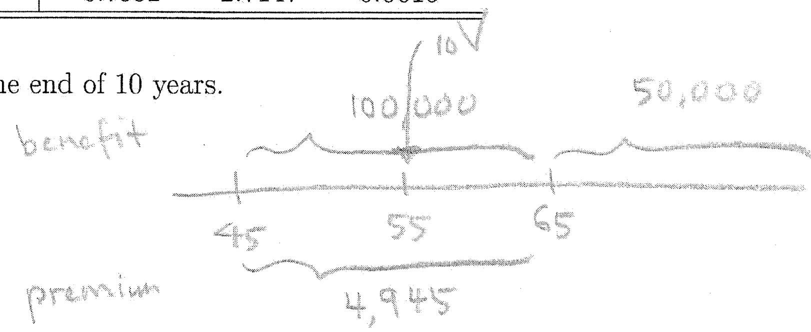
Question No. 3:

For a special whole life insurance on (45), you are given:

- Benefit is paid at the end of the year of death. The death benefit is $100,000 for the first 20 years and reduces to $50,000 thereafter.
- The annual benefit premium of $4,945 is payable once at the beginning of each year for the first 20 years only; no premiums are payable after 20 years.
- The following actuarial present values:

x	A_x	$\ddot{a}_x$	${}_{10}E_x$
55	0.5628	4.8091	0.0758
65	0.7532	2.7147	0.0015

Calculate the benefit reserve at the end of 10 years.



$${}_{10}V = APV(FB_{10}) - APV(FP_{10})$$

$$= \underbrace{100{,}000\left[A_{55} - .5\ {}_{10}E_{55}\ A_{65}\right]}_{53425.37} - \underbrace{4{,}945\left[\ddot{a}_{55} - {}_{10}E_{55}\ \ddot{a}_{65}\right]}_{22\,763.45}$$

$$= \underline{\underline{30{,}661.93}}$$

3

Question No. 5:

For a fully discrete whole life insurance of \$1,000 on (x), you are given:

- The expense, incurred at the beginning of each year, is 10% of the annual benefit premium.
- The gross premium reserve at the end of policy year k is 602.45.
- The gross premium reserve at the end of policy year $k+1$ is 629.72.
- $A_x = 0.6135$
- $i = 5\%$

Calculate q_{x+k}.

$_kV = 602.45$ $\quad$ $_{k+1}V = 629.72$

k $\quad$ $k+1$

$$\text{Benefit Premium} = P = 1000 \frac{A_x}{(1-A_x)/d}$$

$$= 1000 \frac{.6135}{(1-.6135)(.05/1.05)} = 75.58677$$

Since expenses are ~~flat~~/level, $G = 1.10P = 83.14544$

$$_{k+1}V = \frac{(_kV + \overbrace{G - .10P}^{P})(1.05) - 1000q_{x+k}}{1-q_{x+k}}$$

$$629.72(1-q_{x+k}) = (602.45 + (75.58677))(1.05) - 1000q_{x+k}$$

Solving for q_{x+k}, we get

$$q_{x+k} = \frac{[602.45 + (75.58677)](1.05) - 629.72}{1000 - 629.72}$$

$$= \underline{\underline{.222044}}$$

Actual = 385100 * (2044.32 + 2600 * (1-.06))(1.05) - (100,000 - 2324.13)(.0090)
385100

= 1476339836

Expected = 385100 * (2044.32 + 2600 * (1-.05)) (1.02) - 385100 * (100,000 - 2324.13) * .0085

Question No. 6:

An insurer issued 400,000 ~~fully discrete~~ whole life insurance policies to lives all exactly age 50 on January 1, 2002. Each policy issued has a death benefit of \$100,000 with an annual gross premium of \$2,600.

= 1453506616

You are given:

Actual - Expected = 22833220

- The following values in Year 2011:

	anticipated	actual
Expenses as a percent of premium	0.05	0.06
Annual effective rate of interest	0.02	0.05
q_{59}	0.0085	0.0090

- The gross premium reserves per policy at the end of Year 2010 and Year 2011, respectively, are:

$$ {}_9V = 2,044.32 \text{ and } {}_{10}V = 2,324.13 $$

- A total of 385,100 remain in force at the beginning of Year 2011.
- Gains and losses are calculated in the following order: interest then expenses then mortality.

Calculate the total gain (or loss) due to expenses for this portfolio of policies in Year 2011.

385,100

1/1/2011 12/31/2011

Total Gain due to = 385,100 (.05 - .06)(2,600)(1 + .05)

= ~~[illegible]~~ -10,513,230

use 5% — use actual ~~expected~~ interest of .05 since the order starts with interest

Total Gain =
-10513230
+52153939
-18807489
22833220

a loss clearly since actual expenses are larger than anticipated

Gain due to interest = 385100 * (2044.32 + 2600(1-.05))(.05-.02)

52153939

Loss due to mortality = 385100 * (100,000 - 2324.13)(.0085 - .0090)

- 18807489

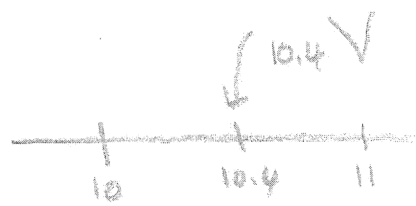
5

Question No. 8:

For a life insurance policy issued to (x), you are given:

$P_{10} = .045$

- Death benefit of \$1 is paid at the end of the year of death.
- The benefit premium in year 11, payable at the beginning of the year, is \$0.045.
- There are no expenses for this policy.
- The policy is still active after 10 years.
- Deaths are assumed to be uniformly distributed over integral ages.
- ${}_{10}V = 0.325$
- $p_{x+10} = 0.925$
- $i = 6\%$



Calculate ${}_{10.4}V$.

$${}_{10.4}V = \frac{({}_{10}V + P_{10})(1+i)^{.4} - {}_{.4}q_{x+10} * v^{-.6}}{1 - {}_{.4}q_{x+10}}$$

By UDD, we have ${}_{h}q_{x+10} = h \cdot q_{x+10}$, $0 \le h \le 1$

$$= \frac{(.325 + .045)(1.06)^{.4} - .4(1-.925)(1.06)^{-.6}}{1 - .4(1-.925)}$$

$$= .360573$$

6

Question No. 1:

The joint lifetime of a husband (x) and a wife (y) is being modeled as:

$$\begin{tikzcd}
\boxed{\begin{matrix}(x) \text{ alive}\\ (y) \text{ alive}\\ 0\end{matrix}} \arrow[r, "\mu^f_{y+t}"] \arrow[d, "\mu^m_{x+t}"] & \boxed{\begin{matrix}(x) \text{ alive}\\ (y) \text{ dead}\\ 1\end{matrix}} \arrow[d, "\mu^m_{x+t}"] \\
\boxed{\begin{matrix}(x) \text{ dead}\\ (y) \text{ alive}\\ 2\end{matrix}} \arrow[r, "\mu^f_{y+t}"] & \boxed{\begin{matrix}(x) \text{ dead}\\ (y) \text{ dead}\\ 3\end{matrix}}
\end{tikzcd}$$

You are given:

$\mu^m_{x+t} = 0.03$, for all $t > 0$

and

$\mu^f_{y+t} = 0.02$, for all $t > 0$

Need ${}_{10}p^{03}_{xy}$ - take the complement

Calculate the probability that (x) and (y), given both are alive today, will be dead within the next 10 years.

$${}_{10}p^{03}_{xy} = 1 - \left({}_{10}p^{00}_{xy} + {}_{10}p^{01}_{xy} + {}_{10}p^{02}_{xy}\right)$$

$$= 1 - \left(e^{-.05(10)} + \int_0^{10} e^{-.05t}\, .02\, e^{-.03(10-t)}\,dt + \int_0^{10} e^{-.05t}\, .03\, e^{-.02(10-t)}\,dt\right)$$

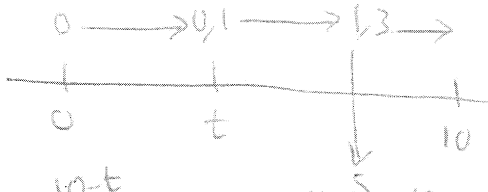
$$= 1 - \left(e^{-.5} + .02\, e^{-.3}\int_0^{10} e^{-.02t}\,dt + .03\, e^{-.2}\int_0^{10} e^{-.03t}\,dt\right)$$

$$= 1 - \left(e^{-.5} + e^{-.3}(1 - e^{-.2}) + e^{-.2}(1 - e^{-.3})\right)$$

$$= 1 - e^{-.3} - e^{-.2} + e^{-.5} = \underline{.0469\,8169}$$

Another solution to Question 1

$0 \to 1 \to 3$

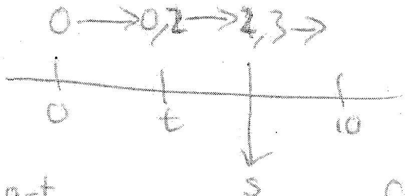


$$\int_0^{10} {}_tp^{00}\mu^{01}\cdot\int_0^{10-t} {}_sp^{11}\mu^{13}\,ds\,dt = \int_0^{10} e^{-.05t}\,.02\int_0^{10-t} e^{-.03s}\,.03\,ds\,dt$$

$$= .02\int_0^{10} e^{-.05t}\left(1-e^{-.03(10-t)}\right)dt$$

$$= \frac{2}{5}\left(1-e^{-.5}\right) - e^{-.3}\left(1-e^{-.2}\right)$$

$0 \to 2 \to 3$



$$\int_0^{10} {}_tp^{00}\mu^{02}\cdot\int_0^{10-t} {}_sp^{22}\mu^{23}\,ds\,dt = \int_0^{10} e^{-.05t}\,.03\int_0^{10-t} e^{-.02s}\,.02\,ds\,dt$$

$$= .03\int_0^{10} e^{-.05t}\left(1-e^{-.02(10-t)}\right)dt$$

$$= \frac{3}{5}\left(1-e^{-.5}\right) - e^{-.2}\left(1-e^{-.3}\right)$$

add the two, we get

$$\left(1-\cancel{e^{-.5}}\right) - e^{-.3} + \cancel{e^{-.5}} - e^{-.2} + e^{-.5} = 1 - e^{-.3} - e^{-.2} + e^{-.5}$$

(same result)

The easy approach is to consider, because of independence and constant force, $T_x \sim$ Exponential $(\mu = .03)$ and $T_y \sim$ Exponential $(\mu = .02)$

$$\Pr[T_x \le 10, T_y \le 10] = \Pr[T_x \le 10] * \Pr[T_y \le 10]$$

$$= \left(1-e^{-.3}\right)\left(1-e^{-.2}\right) = 1 - e^{-.3} - e^{-.2} + e^{-.5}$$

(same result!)