MATH 3631
Actuarial Mathematics II
Final Examination
Tuesday, 7 May 2019
Time Allowed: 2 hours (1:00 - 3:00 pm)
Room: AUST 105
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: Emil **Student ID:** Suggested solutions

- There are twelve (12) written-answer questions here and you are to answer all questions. Each question is worth 10 points. Your final mark will be divided by 120 to convert to a unit of 100%.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.
- Good luck to everyone. Congratulations, and all the best, to those graduating.
- Be safe and enjoy the summer!

Question	Worth	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	
%	÷ 120	

Question No. 1:

For a fully discrete whole life insurance of 10,000 on (50), you are given:

- $q_{59} = 0.020$ and $q_{60} = 0.030$
- $i = 0.05$
- Renewal expenses at the start of each year are 1 plus 1% of the gross premium.
- The gross premium reserve at duration 9 is 1475 and at duration 10 is 1587.

Calculate the annual gross premium.

$$_9V \rightarrow {}_{10}V$$

$$0 \quad 9 \quad 10$$

$$_{10}V = \frac{[_9V + G(1-.01) - 1](1+i) - 10{,}000\, q_{59}}{1 - q_{59}}$$

OR

$$.99G = \frac{1}{1+i}\left[_{10}V(1-q_{59}) + 10{,}000\, q_{59}\right] - {}_9V + 1$$

$$= \underbrace{\frac{1}{1.05}\left[1587(.98) + 10{,}000(.02)\right] - 1475 + 1}_{197.6762}$$

$$G = \frac{197.6762}{.99}$$

$$= 199.6729 \approx \underline{\underline{199.7}} \text{ or } \underline{200}$$

Question No. 2:

An insurance company issues 750 fully discrete whole life insurance policies of \$100,000 to individuals age 55 with independent future lifetimes. You are given:

- The following actual and expected experience in year 11:

Experience	actual	expected
Gross annual premium	\$ 6000	\$ 6000
Maintenance expenses per policy (payable b.o.y.)	425	300
Claim expenses per policy (payable at death)	125	200
q_{65}	0.06	0.04
Annual effective rate of interest	0.062	0.050

- Profits are calculated based on the following (per policy) gross premium reserve at the end of year 10:

$$ {}_{10}V^g = 30,000 $$

- At the end of the 10th year, 516 (of these) insurances remain in force.

Calculate the total gain or loss for the 11th year on this portfolio.

$ {}_{10}V^g \to {}_{11}V^g $

0 10 11

55 65

$$ {}_{11}V^g = \underbrace{\frac{(30,000 + 6000 - 300)(1.05) - (100000 + 200)(.04)}{1-.04}}_{34,871.88} $$

"actual reserve" is

$$ \underbrace{\frac{(30,000 + 6000 - 425)(1.062) - (100,000 + 125)(.06)}{1 - .06}}_{33,801.22} $$

$$ \text{gain/loss} = (33801.22 - 34871.88) \times 516 \text{ policies} $$

$$ = -552,460.6 $$

loss due to

higher maintenance expenses

higher death rates

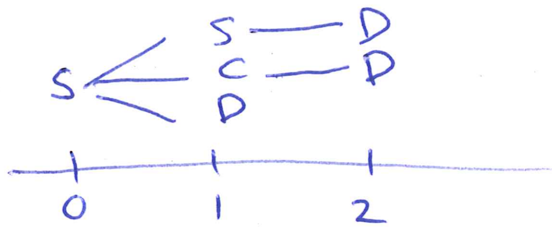
Question No. 3:

Hospital patients are classified as Sick (S), Critical (C), or Discharged (D). Transitions, in days, occur according to the following time-homogeneous transition probability matrix:

$$\begin{array}{c c} & \begin{array}{ccc} S & C & D \end{array} \\ \begin{array}{c} S \\ C \\ D \end{array} & \begin{pmatrix} 0.60 & 0.25 & 0.15 \\ 0.30 & 0.40 & 0.30 \\ 0.00 & 0.00 & 1.00 \end{pmatrix} \end{array}$$

Suppose today that there are exactly 10 sick patients in the hospital. The state of each patient is independent of the state of any other patient.

Calculate the probability that at least 3 of these 10 sick patients will be discharged by the end of two days from today.



$S \to S \to D \qquad .60(0.15) = .09$

$S \to C \to D \qquad .25(.30) = .075$

$S \to D \qquad 0.15$

$.09 + .075 + .15 = .315$

probability ≥ 3 of 10 sick $= 1 - Pr(0) - Pr(1) - Pr(2)$

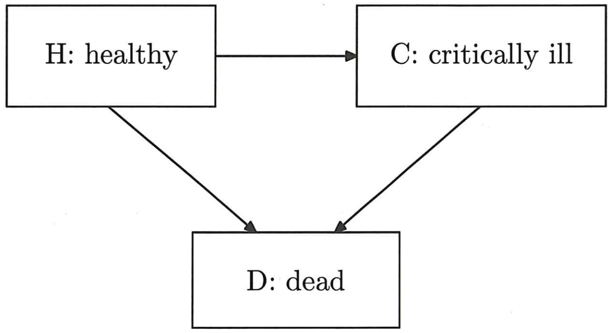
$= 1 - (1-.315)^{10} - 10(.315)(1-.315)^{9} - \binom{10}{2}(.315)^{2}(1-.315)^{8}$

$= 0.6562051 \approx \underline{0.6562}$

Question No. 4:

A life insurer uses the following three-state model to price a two-year critical illness policy issued to healthy policyholders at time $t = 0$:

two years only



You are given:

- The constant forces of transition are:

$$\mu^{HC} = 0.010 \qquad \mu^{HD} = 0.001 \qquad \mu^{CD} = 0.025$$

- The policy pays 10,000 at the moment the policyholder becomes critically ill. No other benefits are provided.
- $\delta = 4\%$

Calculate the actuarial present value of the benefits for this critical illness policy.

10,000

0 t 2

becomes critically ill

$$\text{APV(benefits)} = \int_0^{2} 10{,}000\, e^{-.04t}\, e^{-(.010+.001)t}\, (.010)\, dt$$

$$= 10{,}000(.010) \int_0^{2} e^{-.051t}\, dt$$

$$= 10{,}000\,(.010) * \left(1 - e^{-.051(2)}\right)/.051$$

$$= \underset{\cancel{1960.784}}{190.1381} \approx \underset{\cancel{196.08}}{190}$$

Corrected 5/4/2020

Thx to AB.

Question No. 5:

In a triple decrement table, you are given that decrement (1) is death, decrement (2) is disability, and decrement (3) is withdrawal. You are given:

- $q'^{(1)}_{65} = 0.06$, $q'^{(2)}_{65} = 0.03$ and $q'^{(3)}_{65} = 0.15$.
- Withdrawals occur only at the end of the year.
- Mortality and disability are uniformly distributed over each year of age in the associated single decrement tables.

Calculate $q^{(1)}_{65}$.

UDD $= 1 - t \cdot q'^{(2)}_{65}$

$= 1, \; 0 \leq t < 1$

$$q^{(1)}_{65} = \int_0^1 {}_tp'^{(1)}_{65} \; {}_tp'^{(2)}_{65} \; {}_tp'^{(3)}_{65} \; \mu'^{(1)}_{65+t} \, dt$$

$q'^{(1)}_{65}$ UDD in mortality

$$= q'^{(1)}_{65} \int_0^1 (1 - t \cdot q'^{(2)}_{65}) \, dt$$

$$= .06 \int_0^1 (1 - .03t) \, dt$$

$$= .06 \left[1 - \frac{.03}{2} \right]$$

$$= .0591 \leq .06 = q'^{(1)}_{65}$$ ✓ check

Question No. 6:

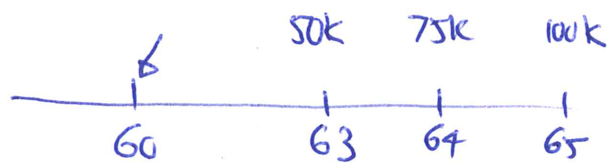
Simon, who is exactly age 60 today, joins XYZ Insurance Company. His annual salary is 250,000 and will remain constant each year. For simplicity, assume that retirement takes place on a birthday.

XYZ offers a pension plan to Simon with the following benefits:

- a retirement benefit equal to 50,000 at time of retirement if he retires at age 63, or 75,000 at time of retirement if he retires at age 64, or 100,000 at time of retirement if he retires at age 65; and
- no other benefits are provided.

Decrements for this pension plan follow the *Standard Service Table* and $i = 0.05$.

Calculate the actuarial present value of his retirement benefits.



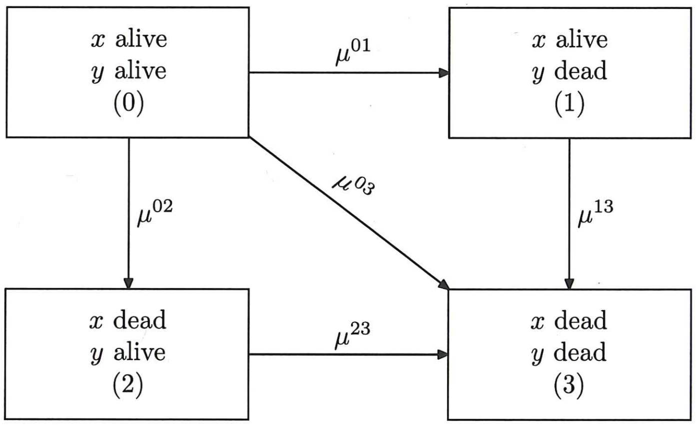
$$APV\left(\text{retirement benefit}\right) = \frac{50{,}000}{\ell_{60}^{(\tau)}}\left[v^3 d_{63}^{(r)} + 1.5 v^4 d_{64}^{(r)} + 2 v^5 d_{65}^{(r)} \right]$$

$$= \frac{50000}{93\,085.4}\left[v^3 (4515.2) + 1.5 v^4 (4061.0) + 2 v^5 (38488.3) \right]$$

$$= \underline{\underline{37{,}183.64}}$$

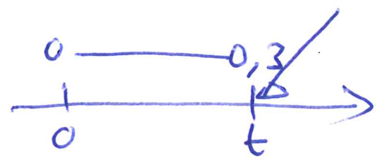
Question No. 7:

The joint mortality of two lines (x) and (y) is being modeled as a multiple state model with a common shock as depicted below:



You are given:

- $\mu^{01} = 0.015$
- $\mu^{02} = 0.025$
- $\delta = 0.035$



A special joint whole life insurance pays 10,000 at the moment of simultaneous death, if that occurs, and zero otherwise. The actuarial present value of this special insurance is 260.

Calculate μ^{03}.

$$\text{APV(special insurance)} = \int_0^\infty 10{,}000 * e^{-.035t}\, e^{-(.015+.025+\mu^{03})t} \cdot \mu^{03}\, dt$$

$$= 10{,}000\mu^{03} \int_0^\infty e^{-(.075+\mu^{03})t}\, dt$$

$$= \frac{10{,}000\,\mu^{03}}{.075+\mu^{03}} = 260$$

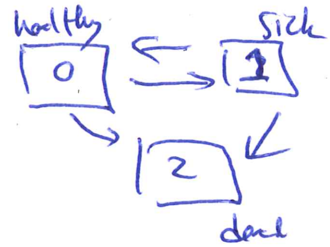
$$\Rightarrow \mu^{03} = \frac{.075(260)}{10{,}000-260} = \underline{\underline{.002}}$$

Question No. 8:

For a whole life insurance policy that provides sickness benefit, you are given: ~~You are given:~~

- The policy is issued to a `Healthy` person age 55.
- Premium is payable continuously at the annual rate of P so long as the policyholder is `Healthy`.
- A sickness benefit of 10,000 per year is payable while the policyholder is Sick.
- A benefit of 200,000 is payable at the moment of `Death` of the policyholder.
- $i = 0.05$
- All probabilities and actuarial functions are based on the *Standard Sickness-Death Model.*

Calculate P.



$$APV(\text{premium}) = P\,\bar{a}_{55}^{00}$$

$$APV(\text{sickness}) = 10{,}000\,\bar{a}_{55}^{01}$$

$$APV(\text{death}) = 200{,}000\,\bar{A}_{55}^{02}$$

$$P = \frac{10{,}000\,\bar{a}_{65}^{01} + 200{,}000\bar{A}_{55}^{02}}{\bar{a}_{55}^{00}}$$

$$= \frac{10{,}000(2.3057) + 200{,}000(.39366)}{10.1228}$$

$$= \frac{101{,}789}{10.1228} = \underline{\underline{10{,}055.42}}$$

Question No. 9:

35 — 65

You are given the following information for Jeff, exact age 35, who just joined a defined benefit pension plan:

- The plan provides a retirement pension of 2% of final average salary for each year of service. The final average salary is defined as the average salary in the three years before retirement.
- His current salary is 65,000 and is expected to increase 4.0% annually on his birthdays.
- He will retire at exact age 65.

35 62 63 64 65

Calculate the replacement ratio provided by Jeff's pension plan.

pension income $= .02 * 30 *$ final avrge lst 3years

$$\frac{1}{3} * 65{,}000 * \left(1.04^{29} + 1.04^{28} + 1.04^{27}\right)$$

$= 117009.4$

final salary at retirement $= 65{,}000\,(1.04)^{29} = 207{,}712.3$

replacement ratio $= \dfrac{117{,}009.4}{207{,}712.3} = 57.72\,\%$

0.5772189

Question No. 10:

A defined benefit pension plan provides participants, upon retirement at age 65, an annual pension equal to 1.5% of final salary per year of service. Final salary is defined to be the salary in the calendar year just prior to retirement. The pension benefit will be paid in the form of a life annuity payable for life at the beginning of each month. Funding method used is Traditional Unit Credit (TUC).

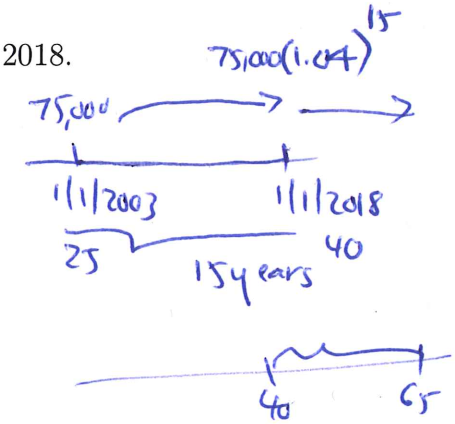
Salaries not projected

Death is the only decrement prior to retirement.

Sheila joined the pension plan at exact age 25 on January 1, 2003, with a then salary of 75,000. She expected this salary to increase by 4% each year on her birthday. As of January 1, 2018, her annual salary has always increased at that rate.

You are given: $i = 0.05$ $\quad {}_{25}p_{40} = 0.75$ $\quad \ddot{a}_{65}^{(12)} = 11.0$

Calculate actuarial liability of Sheila's retirement benefit on January 1, 2018.



$$AB_{40} = .015 * 15 * 75{,}000\,(1.04)^{15}$$

$$AL_{40} = APV(AB_{40})$$

$$= .015 * 15 * 75{,}000\,(1.04)^{15} * v^{25} * {}_{25}p_{40} * \ddot{a}_{65}^{(12)}$$

$$= \underbrace{.015 * 15 * 75{,}000\,(1.04)^{15} * \frac{1}{1.05^{25}} * .75 * 11.0}$$

$$= \underline{\underline{74{,}039.82}}$$

Question No. 12:

For two lives (50) and (60) with independent future lifetimes, you are given:

- ${}_{10}p_{50} = 0.95$
- ${}_{10}p_{\overline{50:60}} = 0.99$

Calculate ${}_{10}p_{50:60}$.

$${}_{10}q_{\overline{50:60}} = \underbrace{{}_{10}q_{50}}_{(1-.95)} \; {}_{10}q_{60} = 1 - .99 = .01$$

$$\therefore \quad {}_{10}q_{60} = \frac{.01}{.05} = .20 \implies {}_{10}p_{60} = .80$$

$${}_{10}p_{50:60} = {}_{10}p_{50} * {}_{10}p_{60}$$

$$= .95 * .80$$

$$= \underline{\underline{0.76}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK