MATH 3631
Actuarial Mathematics II
Final Examination
Monday, 30 April 2018
Time Allowed: 2 hours (6:00 - 8:00 pm)
Room: AUST 110
Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are twelve (12) written-answer questions here and you are to answer all questions. Each question is worth 10 points. Your final mark will be divided by 120 to convert to a unit of 100%.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**
- **Good luck to everyone. Congratulations, and all the best, to those graduating.**
- **Have a safe and enjoyable summer!**

Question	Worth	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	
%	$\div$ 120	

Question No. 1:

For a special 20-year deferred whole life annuity-due of 200 per year issued to (45), you are given:

- Annual premiums are payable for the first 20 years.
- If death occurs during the deferral period, all premiums paid are returned without interest at the end of the year of death.
- Expenses, payable at the beginning of the year, are 10% of premium in the first year and 5% of premiums thereafter.
- $\ddot{a}_{45} = 18.3728$
- $\ddot{a}_{45:\overline{20}|} = 13.1666$
- $(IA)^{\,1}_{45:\overline{20}|} = 0.3014$

Calculate the annual gross premium.

Question No. 2:

For a fully discrete whole life insurance issued to (40), you are given:

- Expenses, incurred at the beginning of each year, consist of 700 in the first year and 300 in subsequent years.
- The gross premium reserve at the end of year 15 is 76,853.
- $\ddot{a}_{40} = 12.85$
- $\ddot{a}_{55} = 8.89$

Calculate the net premium reserve at the end of year 15.

Question No. 3:

For a fully discrete whole life insurance policy of 10,000 on (50), you are given:

(i) The annual gross premium is 120.

(ii) For calculating gross premium reserves in year 6, the following assumptions are made:

- The 5th-year gross premium reserve is ${}_5V^g = 426$.
- $q_{56} = 0.002$
- Annual expenses of 6, payable at the beginning of the year
- $i = 0.06$

(iii) Actual experience during year 6 for this policy is:

- The policy is still inforce and active at the end of year 6.
- The annual expenses are 5, paid at the beginning of the year.
- The actual interest earned is 7.25%.

Calculate the gain (or loss) in year 6 for this one policy.

Question No. 4:

A bank classifies its credit card customers each year according to a three-state time-homogeneous Markov model with states:

(0) preferred

(1) standard

(2) below standard

The one-year transition probabilities are:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & \quad 1 & \quad 2 \end{array} \\ \begin{pmatrix} 0.80 & 0.15 & 0.05 \\ 0.40 & 0.50 & 0.10 \\ 0.00 & 0.25 & 0.75 \end{pmatrix} \end{array}$$

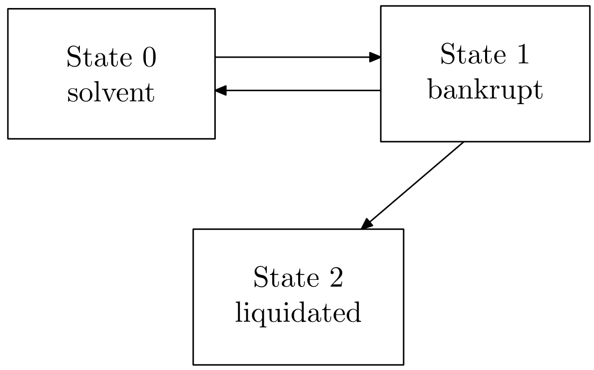
The bank charges an annual fee of 100 payable at the beginning of each year for a customer classified 'standard'. However, this fee is reduced by 20% if the customer moves to a 'preferred' status and is increased by 30% if the customer moves to a 'below standard' status.

Assume that a customer today is classified 'standard' and will keep its credit relationship with the bank for the next three years. Interest rate is $i = 0.05$.

Calculate the actuarial present value of the fees to be paid by this customer in the next three years.

Question No. 5:

Beginning at time 0, the financial strength of a company is based on the following Markov model:



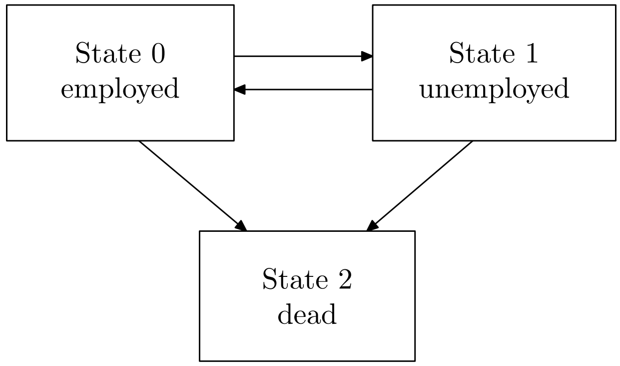
You are given:

- $\mu_t^{12} = 0.0018$ at $t = 8$
- $\dfrac{d}{dt}\,{}_tp^{10} = 0.0015$ at $t = 8$
- $\dfrac{d}{dt}\,{}_tp^{11} = -0.0030$ at $t = 8$

Calculate ${}_tp^{11}$ at $t = 8$.

Question No. 6:

You are given the following Markov model:



All transition intensities are constant and independent of age:

$$\mu^{01} = 0.010, \quad \mu^{02} = 0.004, \quad \mu^{10} = 0.025, \quad \text{and} \quad \mu^{12} = 0.075$$

Calculate the probability that during the next 10 years, an 'employed' will become 'unemployed' exactly once and remain 'unemployed' until the end of 10 years.

Question No. 7: (revised version)

You are given:

- The following information from a double decrement table:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{\prime(1)}$	$q_x^{\prime(2)}$
45	0.17	0.08	0.18	–
46	–	–	0.20	–

- $\ell_{45}^{(\tau)} = 100000$
- $q_{46}^{\prime(2)} = \dfrac{5}{3} \times q_{45}^{\prime(2)}$

Calculate $\ell_{47}^{(\tau)}$.

Question No. 8:

For a Type A Universal Life policy with a total death benefit of 150,000, you are given:

policy year	annual premium deposit	percent of premium charge	annual fixed expense charge	annual cost of insurance rate per 1,000	interest credited
1	3000	10%	30	2.5	5%
2	4000	5%	30	3.0	5%

Calculate the account value at the end of the second year.

Question No. 9:

For a Type B universal life insurance policy with an Additional Death Benefit (ADB) of 125,000, you are given:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Annual Cost of Insurance (COI) Rate	Annual Credited Interest Rate
1	1000	2%	25	0.005	5.0%
2	2000	f	50	0.006	5.0%
3	5000	f	75	0.007	5.0%

The account value at the end of year 3 is 5,814.17.

Calculate f.

Question No. 10:

You are given:

- Male mortality is based on a constant force of mortality with $\mu = 0.01$.
- Female mortality follows DeMoivre's law with $\omega = 120$.
- Assume for any pairs of male and female, their future lifetimes are independent.

Calculate the probability that a female age 50 will outlive a male age 50 by at least 5 years.

Question No. 11:

For a last-survivor whole life insurance issued to two lives, each both age 55, you are given:

- The policy pays a death benefit of B at the end of the year of the second death.
- Annual level premiums of 1,000 are paid at the beginning of each year that at least one of the two lives is alive.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$
- The future lifetimes of the two lives are independent.

Calculate B.

Question No. 12:

For two lives both of the same age 50, you are given:

- They have independent future lifetimes.
- Mortality for both lives follows DeMoivre's law with $\omega = 100$.
- $\delta = 0.05$

Calculate $\bar{A}_{\overline{50:50}}$.

Hint: $\int te^{-kt}dt = -\frac{1}{k}e^{-kt}\left(t + \frac{1}{k}\right)$, for $k > 0$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK