MATH 3631

Actuarial Mathematics II

Final Examination

Monday, 30 April 2018

Time Allowed: 2 hours (6:00 - 8:00 pm)

Room: AUST 110

Total Marks: 120 points

Please write your name and student number at the spaces provided:

Name: Emil

Student ID: Suggested Solutions

- There are twelve (12) written-answer questions here and you are to answer all questions. Each question is worth 10 points. Your final mark will be divided by 120 to convert to a unit of 100%.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.
- Good luck to everyone. Congratulations, and all the best, to those graduating.
- Have a safe and enjoyable summer!

Question	Worth	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
Total	120	
%	÷ 120	

Question No. 1:

For a special 20-year deferred whole life annuity-due of 200 per year issued to (45), you are given:

- Annual premiums are payable for the first 20 years.
- If death occurs during the deferral period, all premiums paid are returned without interest at the end of the year of death.
- Expenses, payable at the beginning of the year, are 10% of premium in the first year and 5% of premiums thereafter.
- $\ddot{a}_{45} = 18.3728$
- $\ddot{a}_{45:\overline{20}|} = 13.1666$
- $(IA)^{1}_{45:\overline{20}|} = 0.3014$

$\ddot{a}_{45} - \ddot{a}_{45:\overline{20}|} = {}_{20}E_{45}\, \ddot{a}_{65} = 5.2062$

$G = ?$

Calculate the annual gross premium.

APV(premiums) = APV(benefits) + APV(expenses)

$$G\,\ddot{a}_{45:\overline{20}|} = 200\ {}_{20}E_{45}\,\ddot{a}_{65} + G\,(IA)^{1}_{45:\overline{20}|} + .05G + .05G\,\ddot{a}_{45:\overline{20}|}$$

$$G\left(.95\,\ddot{a}_{45:\overline{20}|} - .05 - (IA)^{1}_{45:\overline{20}|}\right) = 200\ {}_{20}E_{45}\,\ddot{a}_{65}$$

Solving for G, we get

$$G = \frac{200(5.2062)}{.95(13.1666) - .05 - .3014}$$

$$= \frac{1041.24}{12.15687} = \underline{\underline{85.65034}}$$

Question No. 2:

For a fully discrete whole life insurance issued to (40), you are given:

- Expenses, incurred at the beginning of each year, consist of 700 in the first year and 300 in subsequent years.
- The gross premium reserve at the end of year 15 is 76,853.
- $\ddot{a}_{40} = 12.85$
- $\ddot{a}_{55} = 8.89$

Calculate the net premium reserve at the end of year 15.

$$ {}_{15}V^e = \frac{-400}{\ddot{a}_{40}} \ddot{a}_{55} = \frac{-400\,(8.89)}{12.85} = -276.7315 $$

$$ {}_{15}V^n = {}_{15}V^g - {}_{15}V^e $$

$$ = 76853 + 276.7315 $$

$$ = 77{,}129.73 $$

Question No. 3:

For a fully discrete whole life insurance policy of 10,000 on (50), you are given:

(i) The annual gross premium is 120.

(ii) For calculating gross premium reserves in year 6, the following assumptions are made:

- The 5th-year gross premium reserve is ${}_5V^g = 426$.
- $q_{56} = 0.002$
- Annual expenses of 6, payable at the beginning of the year
- $i = 0.06$

(iii) Actual experience during year 6 for this policy is:

- The policy is still inforce and active at the end of year 6.
- The annual expenses are 5, paid at the beginning of the year.
- The actual interest earned is 7.25%.

Calculate the gain (or loss) in year 6 for this one policy.

$$ {}_6V^g = \frac{(426+120-6)(1.06) - 10000(.002)}{1-.002} = 553.507 $$

"actual reserve" is

$$ (426+120-5)(1.0725) = 580.2225 $$

$$ \text{gain/loss} = 580.2225 - 553.507 = \underline{\underline{26.71549}} $$

Question No. 4:

A bank classifies its credit card customers each year according to a three-state time-homogeneous Markov model with states:

(0) preferred

(1) standard

(2) below standard

$(0\ 1\ 0) * Q = (.4\ \ .5\ \ .1)$

$(.4\ .5\ .1) * Q = (.52\ \ .335\ \ .145)$

The one-year transition probabilities are:

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.80 & 0.15 & 0.05 \\ 0.40 & 0.50 & 0.10 \\ 0.00 & 0.25 & 0.75 \end{pmatrix} \end{array}$$

The bank charges an annual fee of 100 payable at the beginning of each year for a customer classified 'standard'. However, this fee is reduced by 20% if the customer moves to a 'preferred' status and is increased by 30% if the customer moves to a 'below standard' status.

Assume that a customer today is classified 'standard' and will keep its credit relationship with the bank for the next three years. Interest rate is $i = 0.05$.

Calculate the actuarial present value of the fees to be paid by this customer in the next three years.

Probabilities × CF

Year 0: 100

Year 1: $80(.4) + 100(.5) + 130(.1) = 95$

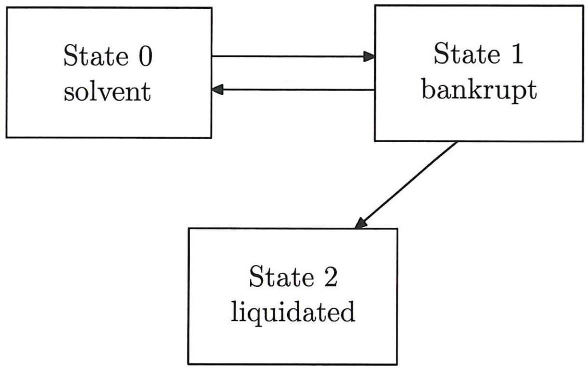
Year 2: $80(.52) + 100(.335) + 130(.145) = 93.95$

$$APV = 100 + 95 * \frac{1}{1.05} + 93.95 * \frac{1}{1.05^2}$$

$$= \underline{\underline{275.6916}}$$

Question No. 5:

Beginning at time 0, the financial strength of a company is based on the following Markov model:



You are given:

- $\mu_t^{12} = 0.0018$ at $t = 8$
- $\dfrac{d}{dt}\,{}_tp^{10} = 0.0015$ at $t = 8$
- $\dfrac{d}{dt}\,{}_tp^{11} = -0.0030$ at $t = 8$

Calculate ${}_tp^{11}$ at $t = 8$.

$$\frac{d}{dt}\,{}_tp^{10} = \cancel{{}_tp^{11}\mu^{10}} - \cancel{{}_tp^{10}}\,\mu^{01}$$

$$+)\quad \frac{d}{dt}\,{}_tp^{11} = \cancel{{}_tp^{10}\mu^{01}} - {}_tp^{11}(\cancel{\mu^{10}} + \mu^{12})$$

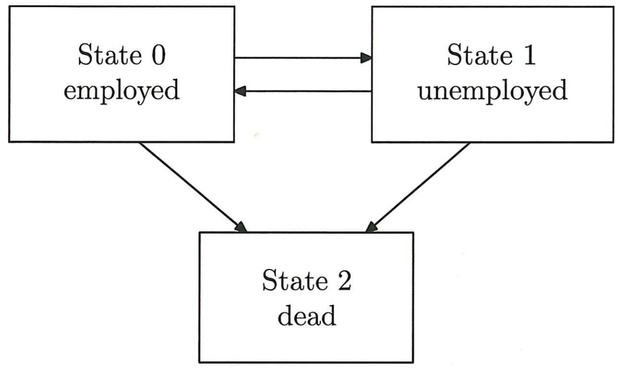
Sum at $t=8$,

$$\underbrace{.0015 - .0030}_{-.0015} = -{}_tp^{11}(.0018)$$

$${}_tp^{11} = \frac{.0015}{.0018} = .8\overline{333}$$

Question No. 6:

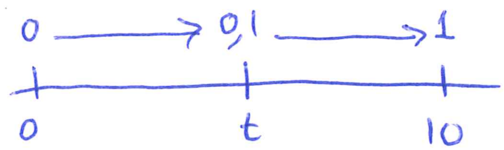
You are given the following Markov model:



All transition intensities are constant and independent of age:

$$\mu^{01} = 0.010, \quad \mu^{02} = 0.004, \quad \mu^{10} = 0.025, \quad \text{and} \quad \mu^{12} = 0.075$$

Calculate the probability that during the next 10 years, an 'employed' will become 'unemployed' exactly once and remain 'unemployed' until the end of 10 years.



$$\int_0^{10} {}_tp^{\overline{00}}\, \mu^{01}\, {}_{10-t}p^{\overline{11}}\, dt$$

$$= \int_0^{10} e^{-.014t}\,(.01)\, e^{-.10(10-t)}\, dt$$

$$= .01 e^{-1} \int_0^{10} e^{+.086t}$$

$$= \frac{.01}{.086} e^{-1} (e^{+.86}-1)$$

$$= \cancel{.0360619} \quad .0583149$$ ok

Question No. 7: (revised version)

You are given:

- The following information from a double decrement table:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{\prime(1)}$	$q_x^{\prime(2)}$
45	0.17	0.08	0.18	–
46	–	–	0.20	–

- $\ell_{45}^{(\tau)} = 100000$
- $q_{46}^{\prime(2)} = \frac{5}{3} \times q_{45}^{\prime(2)}$ $= \frac{5}{3}(.08537) = 0.14228$

Calculate $\ell_{47}^{(\tau)}$.

$p_{45}^{(\tau)} = 1-(.17+.08) = .75 \Rightarrow \ell_{46}^{(\tau)} = \underbrace{.75(100000)}_{75,000}$

$\Downarrow$

$p_{45}^{\prime(1)} p_{45}^{\prime(2)} = .82\, p_{45}^{\prime(2)} = .75 \Rightarrow p_{45}^{\prime(2)} = \frac{.75}{.82}$

$\Rightarrow q_{45}^{\prime(2)} = 0.08537$

$p_{46}^{(\tau)} = p_{46}^{\prime(1)} p_{46}^{\prime(2)} = .8(1-.14228) = 0.6861789$

$\ell_{47}^{(\tau)} = 75000(0.6861789) = 51,463.41$

Question No. 8:

For a Type A Universal Life policy with a total death benefit of 150,000, you are given:

policy year	annual premium deposit	percent of premium charge	annual fixed expense charge	annual cost of insurance rate per 1,000	interest credited
1	3000	10%	30	2.5	5%
2	4000	5%	30	3.0	5%

Calculate the account value at the end of the second year.

$$AV_1 = \frac{(3000(1-.10)-30)(1.05) - 150000 * 2.5/1000}{1 - 2.5/1000}$$

$$= 2434.586$$

$$AV_2 = \frac{(AV_1 + 4000(.95) - 30)(1.05) - 150000 * 3/1000}{1 - 3/1000}$$

$$= \underline{\underline{6083.065}}$$

Question No. 9:

For a Type B universal life insurance policy with an Additional Death Benefit (ADB) of 125,000, you are given:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Annual Cost of Insurance (COI) Rate	Annual Credited Interest Rate
1	1000	2%	25	0.005	5.0%
2	2000	f	50	0.006	5.0%
3	5000	f	75	0.007	5.0%

The account value at the end of year 3 is 5,814.17.

Calculate f.

$$AV_1 = (1000(.98) - 25)(1.05) - 125000(.005)$$

$$= 377.75$$

$$AV_2 = (AV_1 + 2000(1-f) - 50)(1.05) - 125000(.006)$$

$$= 1694.138 - 2100f$$

$$AV_3 = (1694.138 - 2100f + 5000(1-f) - 75)(1.05) - 125000(.007)$$

$$= 6075.094 - 7455f = 5814.17$$

Solving for f, we get

$$f = \frac{6075.094 - 5814.17}{7455} = \underline{\underline{.035}}$$

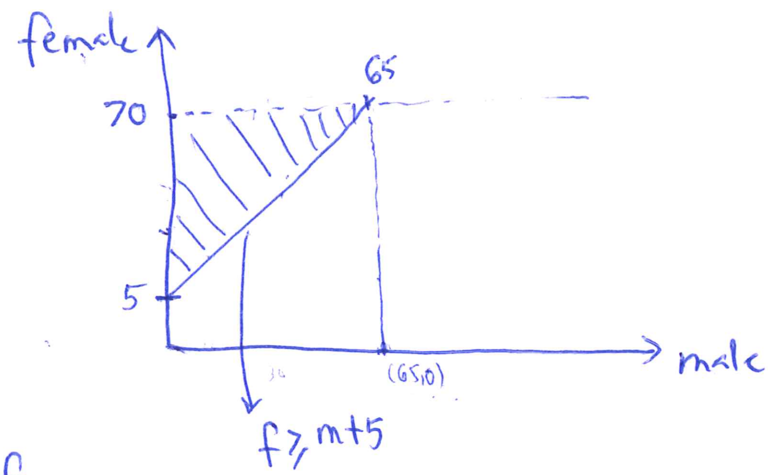
Question No. 10:

You are given:

- Male mortality is based on a constant force of mortality with $\mu = 0.01$. $T_{50}^{m} \sim \text{exponential}(.01)$
- Female mortality follows DeMoivre's law with $\omega = 120$. $T_{50}^{f} \sim \text{uniform}(0,70)$
- Assume for any pairs of male and female, their future lifetimes are independent.

Calculate the probability that a female age 50 will outlive a male age 50 by at least 5 years.

$$\Pr\left(T_{50}^{f} \geq T_{50}^{m} + 5\right)$$



$$\int_5^{70} \int_0^{f-5} \frac{1}{70}\, .01\, e^{-.01m}\, dm\, df$$

$$\frac{1}{70} \int_5^{70} \left(1 - e^{-.01(f-5)}\right) df$$

$$\frac{1}{70}\left(65 - e^{.05}\, \frac{1}{.01}\left(e^{-.05} - e^{-.70}\right)\right)$$

$$\frac{1}{70}\left(65 - 100\left(1 - e^{-.65}\right)\right)$$

$$\frac{1}{70}\left(100 e^{-.65} - 35\right) = \underline{\underline{.2457797}}$$

Question No. 11:

For a last-survivor whole life insurance issued to two lives, each both age 55, you are given:

- The policy pays a death benefit of B at the end of the year of the second death.
- Annual level premiums of 1,000 are paid at the beginning of each year that at least one of the two lives is alive.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$
- The future lifetimes of the two lives are independent.

Calculate B.

APV(premium) = APV(benefit)

$$1000\, \ddot{a}_{\overline{55:55}} = B\, A_{\overline{55:55}}$$

$$B = \frac{1000\left(2\ddot{a}_{55} - \ddot{a}_{55:55}\right)}{2A_{55} - A_{55:55}}$$

$\ddot{a}_{55} = 12.2758$

$\ddot{a}_{55:55} = 10.4720$

$A_{55} = .30514$

$A_{55:55} = .40724$

$$= \frac{14079.6}{0.20304}$$

$$= 69{,}343.97$$

Question No. 12:

For two lives both of the same age 50, you are given:

- They have independent future lifetimes.
- Mortality for both lives follows DeMoivre's law with $\omega = 100$.
- $\delta = 0.05$

$T_{50} \sim$ uniform, density $= \frac{1}{50}$

Calculate $\bar{A}_{\overline{50:50}}$.

Hint: $\int te^{-kt}dt = -\frac{1}{k}e^{-kt}\left(t+\frac{1}{k}\right)$, for $k>0$

$$F_{T_{\overline{50:50}}}(t) = \Pr(T_{50} \le t)\Pr(T_{50} \le t) = (t/50)^2 = t^2/2500, \quad 0 \le t < 50$$

$$f_{T_{\overline{50:50}}}(t) = 2t/2500, \quad 0 \le t < 50$$

$$\bar{A}_{\overline{50:50}} = \int_0^{50} e^{-.05t}\, 2t/2500\, dt$$

$$= \frac{2}{2500}\underbrace{\int_0^{50} te^{-.05t}dt}_{-\frac{1}{.05}e^{-.05t}\left(t+\frac{1}{.05}\right)\Big|_0^{50}}$$

$$= \frac{2}{2500}(-20)\Big(70\underbrace{e^{-.05(50)}}_{e^{-2.5}} - 20\Big)$$

$$= -\frac{4}{250}\left(70e^{-2.5} - 20\right) = \underline{\underline{0.2280648}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK