MATH 3631
Actuarial Mathematics II
Class Test 2 - 3:35-4:50 PM
Monday, 15 April 2019
Time Allowed: 1 hour and 15 minutes
Total Marks: 100 points

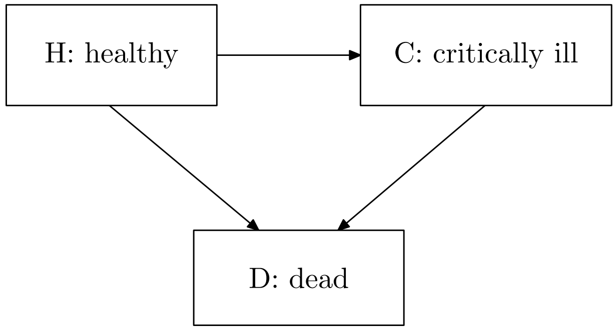
Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

A critical illness model is depicted according to the following multiple state model:



You are the following forces of transition for a life (x):

$$\mu_{x+t}^{\text{HC}} = \begin{cases} 0.02, & t \leq 5 \\ 0.04, & t > 5 \end{cases}$$

$$\mu_{x+t}^{\text{HD}} = 0.01, \quad \text{for all } t$$

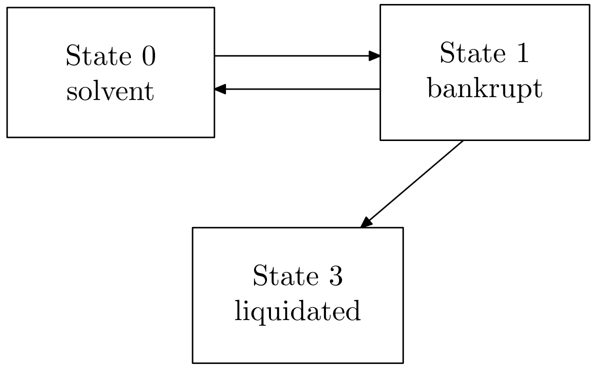
and

$$\mu_{x+t}^{\text{CD}} = 0.05, \quad \text{for all } t$$

Calculate the probability that a healthy life (x) will remain continuously healthy for the following 10 years.

Question No. 2:

The financial strength of a company is based on the following Markov model:



You are given the constant forces of transition:

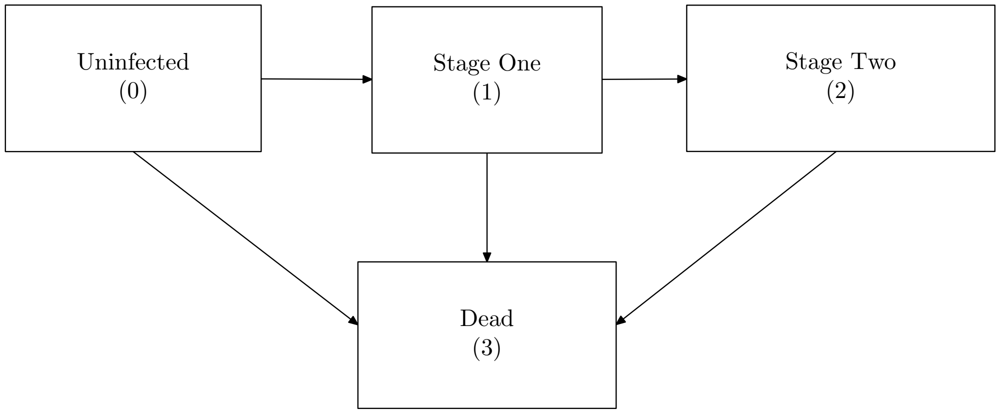
$$\mu^{01} = 0.01 \qquad \mu^{10} = 0.05 \qquad \mu^{12} = 0.25$$

In calculating transition probabilities, use the Kolmogorov's forward equations with the Euler approximation based on steps of $h = 0.5$.

Calculate the probability that a company currently *bankrupt* will be *solvent* at the end of one year.

Question No. 3:

A disease progresses according to the following multiple state model:



An insurance company provides coverage for this disease by paying a fixed benefit amount of 15,000 at the moment an uninfected policyholder reaches 'Stage Two'. You are given:

- All transition intensities are constant and independent of age:

$$\mu^{01} = 0.002, \quad \mu^{12} = 0.100, \quad \mu^{03} = 0.004, \quad \mu^{13} = 0.050, \quad \text{and} \quad \mu^{23} = 0.250$$

- $\delta = 0.05$

Calculate the actuarial present value for this insurance benefit.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Question No. 4:

A bank classifies its credit card customers each year according to a three-state time-homogeneous Markov model with states:

(0) preferred

(1) standard

(2) below standard

The one-year transition probabilities are:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0.85 & 0.10 & 0.05 \\ 0.40 & 0.50 & 0.10 \\ 0.00 & 0.20 & 0.80 \end{pmatrix} \end{array}$$

The bank charges an annual fee of 100 payable at the beginning of each year for a customer classified 'standard'. However, this fee is reduced by 20% if the customer moves to a 'preferred' status and is increased by 30% if the customer moves to a 'below standard' status.

Assume that a customer today is classified 'standard' and will keep its credit relationship with the bank for the next three years. Interest rate is $i = 0.05$.

Calculate the actuarial present value of the fees to be paid by this customer in the next three years.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Question No. 5:

You are given the following extract from a triple-decrement table:

age x	no. of lives $\ell_x^{(\tau)}$	heart disease $q_x^{(1)}$	accidents $q_x^{(2)}$	other causes $q_x^{(3)}$
50	$1,000,000$	0.0008	0.0010	0.0015
51	–	0.0010	0.0015	0.0018
52	–	0.0012	0.0020	0.0020

Calculate $\ell_{52}^{(\tau)}$.

Question No. 6:

You are given the following extract from a triple decrement model:

x	$\ell_x^{(\tau)}$	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
55	-	0.02	0.05	0.06
56	8800	0.05	0.10	0.20

Suppose that after preparing this table, you corrected that $q_{55}^{(3)}$ should have been 0.05 while all other probability values in the table above remain the same.
Calculate the correct value of $d_{56}^{(3)}$.

Question No. 7:

In a triple decrement table, you are given that decrement (1) is death, decrement (2) is disability, and decrement (3) is withdrawal. You are given:

- $q_{65}^{\prime(1)} = 0.05$, $q_{65}^{\prime(2)} = 0.02$ and $q_{65}^{\prime(3)} = 0.10$.
- Withdrawals occur only at the end of the year.
- Mortality and disability are uniformly distributed over each year of age in the associated single decrement tables.

Calculate $p_{65}^{(\tau)}$.

Question No. 8:

You are given:

- An insurance policy issued to (50) will pay $50,000$ upon death if death is accidental and occurs within 15 years.
- An additional benefit of $10,000$ will be paid regardless of the time or cause of death.
- The force of accidental death at all ages is 0.001.
- The force of death for all other causes is 0.005 at all ages.
- $\delta = 0.05$

Calculate the actuarial present value for this policy.

Question No. 9:

You are given the joint density of (T_x, T_y):

$$f_{T_xT_y}(s,t) = \frac{1}{90}(s+t), \quad \text{for } 0 < s < 4, \ 0 < t < 5.$$

Calculate the probability (x) outlives (y).

Question No. 10:

You are given:

- ${}_{10|}q_{\overline{50:60}} = 0.0105$
- ${}_{10}p_{50} = 0.800$
- ${}_{10}p_{60} = 0.750$
- $p_{60} = 0.975$

Calculate q_{70}.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK