MATH 3631

Actuarial Mathematics II

Class Test 2 - 3:35-4:50 PM

Monday, 15 April 2019

Time Allowed: 1 hour and 15 minutes

Total Marks: 100 points

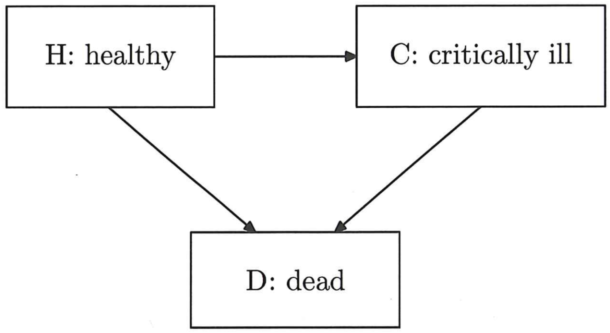
Please write your name and student number at the spaces provided:

Name: Emil Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

A critical illness model is depicted according to the following multiple state model:



You are the following forces of transition for a life (x):

$$\mu_{x+t}^{\text{HC}} = \begin{cases} 0.02, & t \leq 5 \\ 0.04, & t > 5 \end{cases}$$

$$\mu_{x+t}^{\text{HD}} = 0.01, \quad \text{for all } t$$

and

$$\mu_{x+t}^{\text{CD}} = 0.05, \quad \text{for all } t$$

Calculate the probability that a healthy life (x) will remain continuously healthy for the following 10 years.

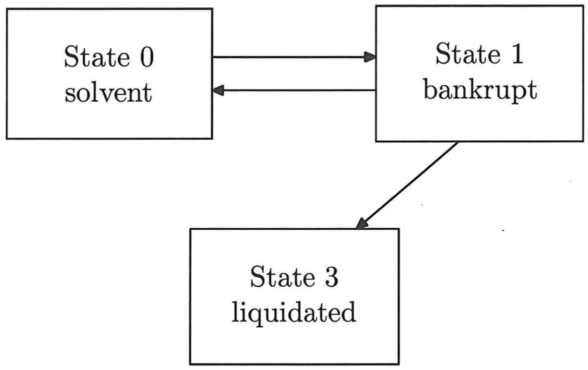
$${}_{10}p_x^{\overline{HH}} = e^{-\int_0^{10}(\mu_{x+t}^{HC} + \mu_{x+t}^{HD})dt}$$

$$= e^{-.02(5) - .04(5) - .01(10)}$$

$$= e^{-0.4} = 0.67032$$

Question No. 2:

The financial strength of a company is based on the following Markov model:



You are given the constant forces of transition:

$$\mu^{01} = 0.01 \qquad \mu^{10} = 0.05 \qquad \mu^{12} = 0.25$$

In calculating transition probabilities, use the Kolmogorov's forward equations with the Euler approximation based on steps of $h = 0.5$.

Calculate the probability that a company currently *bankrupt* will be *solvent* at the end of one year.

Applying KFE, we get $\frac{d}{dt}\,{}_tp^{10} = {}_tp^{11}\mu^{10} - {}_tp^{10}(\mu^{01} + \cancel{\mu^{02}})$ 0

and $\frac{d}{dt}\,{}_tp^{12} = {}_tp^{11}\mu^{12}$

Apply Euler's approximation with $h = 0.5$:

at $t=0$, ${}_0p^{10} = {}_0p^{12} = 0$, ${}_0p^{11} = 1$, so that

$${}_{0.5}p^{10} \approx \cancel{{}_0p^{10}}^{\,0} + 0.5\left[\underbrace{{}_0p^{11}}_{1}\underbrace{\mu^{10}}_{0.05} - \cancel{{}_0p^{10}}^{\,0}(\mu^{01} + \cancel{\mu^{02}}^{\,0})\right] = .025$$

$${}_{0.5}p^{12} \approx \cancel{{}_0p^{12}}^{\,0} + 0.5\,\underbrace{{}_0p^{11}}_{1}\underbrace{\mu^{12}}_{.25} = 0.5(.25) = .125$$

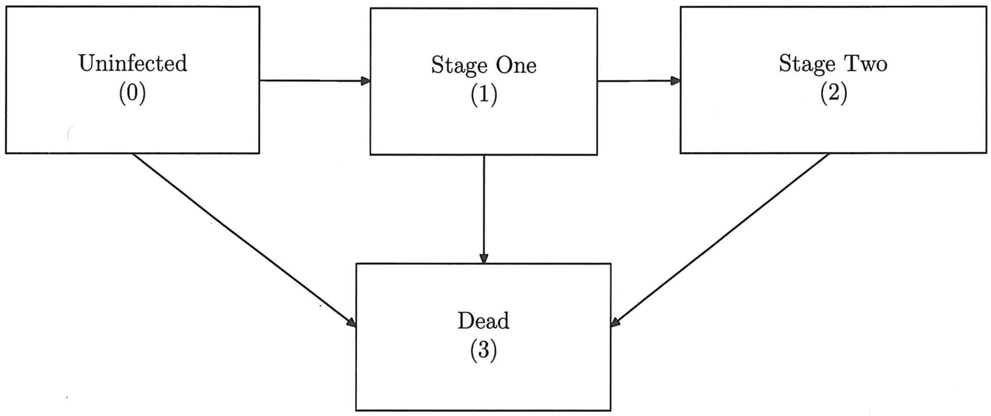
$${}_{0.5}p^{11} = 1 - {}_{.5}p^{10} - {}_{.5}p^{12} = 1 - .025 - .125 = .850$$

at $t = 0.5$, $h = 0.5$, we have

$${}_1p^{10} = {}_{0.5}p^{10} + 0.5\left[{}_{.5}p^{11}\mu^{10} - {}_{.5}p^{10}(\mu^{01} + \cancel{\mu^{02}}^{\,0})\right] = .025 + \overbrace{.5[.85(.05) - .025(.01)]}^{0.046125}$$

Question No. 3:

A disease progresses according to the following multiple state model:



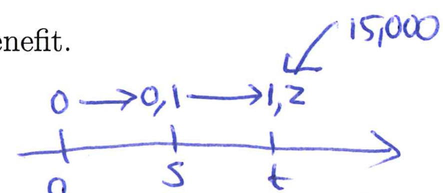
An insurance company provides coverage for this disease by paying a fixed benefit amount of 15,000 at the moment an uninfected policyholder reaches 'Stage Two'. You are given:

- All transition intensities are constant and independent of age:

$$\mu^{01} = 0.002, \quad \mu^{12} = 0.100, \quad \mu^{03} = 0.004, \quad \mu^{13} = 0.050, \quad \text{and} \quad \mu^{23} = 0.250$$

- $\delta = 0.05$

Calculate the actuarial present value for this insurance benefit.



$$APV(0 \to 2) = \int_0^\infty 15000\, e^{-.05t} \int_0^t e^{-.006s} (.002)\, e^{-.15(t-s)} (.10)\, ds\, dt$$

$$= 15000(.002)(.10) \int_0^\infty e^{-.20t} \int_0^t e^{+.144s}\, ds\, dt$$

$$= 15000(.002)(.10) \int_0^\infty e^{-.20t} \frac{1}{.144} \left(e^{.144t} - 1\right) dt$$

$$= \frac{15000(.002)(.10)}{.144} \int_0^\infty \left(e^{-.056t} - e^{-.20t}\right) dt$$

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$$= \frac{15000(.002)(.10)}{.144}\left(\frac{1}{.056} - \frac{1}{.20}\right)$$

$$= \underline{\underline{267.8571}}$$

Question No. 4:

A bank classifies its credit card customers each year according to a three-state time-homogeneous Markov model with states:

(0) preferred

(1) standard

(2) below standard

$(0\ 1\ 0) * Q = (.4\ .5\ .1)$

$(.4\ .5\ .1) * Q = (.54\ .31\ .15)$

The one-year transition probabilities are:

$$Q = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0.85 & 0.10 & 0.05 \\ 0.40 & 0.50 & 0.10 \\ 0.00 & 0.20 & 0.80 \end{pmatrix} \end{array}$$

The bank charges an annual fee of 100 payable at the beginning of each year for a customer classified 'standard'. However, this fee is reduced by 20% if the customer moves to a 'preferred' status and is increased by 30% if the customer moves to a 'below standard' status.

Assume that a customer today is classified 'standard' and will keep its credit relationship with the bank for the next three years. Interest rate is $i = 0.05$.

Calculate the actuarial present value of the fees to be paid by this customer in the next three years.

Probabilities * Cash Flow

Year 0: 100

Year 1: $80(.4) + 100(.5) + 130(.1) = 95$

Year 2: $80(.54) + 100(.31) + 130(.15) = 93.70$

$$APV = 100 + 95\frac{1}{1.05} + 93.70\frac{1}{1.05^2} = \underline{\underline{275.4649}}$$

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Question No. 5:

You are given the following extract from a triple-decrement table:

age x	no. of lives $\ell_x^{(\tau)}$	heart disease $q_x^{(1)}$	accidents $q_x^{(2)}$	other causes $q_x^{(3)}$	
50	1,000,000	0.0008	0.0010	0.0015	$= .0033$
51	—	0.0010	0.0015	0.0018	$= .0043$
52	—	0.0012	0.0020	0.0020	$= .0052$

Calculate $\ell_{52}^{(\tau)}$.

$\ell_{51}^{(\tau)} = 1000000 * (1 - .0033) = 996{,}700$

$\ell_{52}^{(\tau)} = 996{,}700 \times (1 - .0043) = \underline{\underline{992{,}414.2}}$

Question No. 6:

You are given the following extract from a triple decrement model:

$.02 + .05 + .06 = .13$

x	$\ell_x^{(\tau)}$	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
55	-	0.02	0.05	0.06
56	8800	0.05	0.10	0.20

Suppose that after preparing this table, you corrected that $q_{55}^{(3)}$ should have been 0.05 while all other probability values in the table above remain the same.
Calculate the correct value of $d_{56}^{(3)}$.

$$l_{56}^{(\tau)} = l_{55}^{(\tau)}(1 - .13) \Rightarrow l_{55}^{(\tau)} = \frac{8800}{.87} = 10114.94$$

$$\text{new } l_{56}^{(\tau)} = 10114.94(1 - .02 - .05 - .05) = 8901.149$$

$$\text{Correct value of } d_{56}^{(3)} = \text{new } l_{56}^{(\tau)} * .20$$

$$= 8901.149 * .20$$

$$= \underline{\underline{1780.23}}$$

Question No. 7:

In a triple decrement table, you are given that decrement (1) is death, decrement (2) is disability, and decrement (3) is withdrawal. You are given:

- $q'^{(1)}_{65} = 0.05$, $q'^{(2)}_{65} = 0.02$ and $q'^{(3)}_{65} = 0.10$.
- Withdrawals occur only at the end of the year.
- Mortality and disability are uniformly distributed over each year of age in the associated single decrement tables.

Calculate $p^{(\tau)}_{65}$.

$$p^{(\tau)}_{65} = p'^{(1)}_{65}\, p'^{(2)}_{65}\, p'^{(3)}_{65}$$

since $p^{(\tau)}_{65} = e^{-\int_0^1 (\mu^{(1)}_{65+t} + \mu^{(2)}_{65+t} + \mu^{(3)}_{65+t})dt}$

$\mu'^{(1)} = \mu^{(1)} \ldots$

$$= .95(.98)(.90)$$

$$= \underline{\underline{.8379}}$$

Question No. 8:

You are given:

- An insurance policy issued to (50) will pay 50,000 upon death if death is accidental and occurs within 15 years.
- An additional benefit of 10,000 will be paid regardless of the time or cause of death.
- The force of accidental death at all ages is 0.001.
- The force of death for all other causes is 0.005 at all ages.
- $\delta = 0.05$

Calculate the actuarial present value for this policy.

$$APV(policy) = \int_0^{15} 50000\, e^{-.05t}\, \underbrace{{}_tp_{50}^{(\tau)}}_{e^{-.006t}}\, \underbrace{\mu_{50+t}^{(acc)}}_{.001}\, dt + \int_0^{\infty} 10{,}000\, e^{-.05t}\, \underbrace{{}_tp_{50}^{(\tau)}}_{e^{-.006t}}\, \underbrace{\mu_{50+t}^{(\tau)}}_{.006}\, dt$$

$$= \frac{50000(.001)}{.056}\left(1 - e^{-.056(15)}\right) + \frac{10000(.006)}{.056}$$

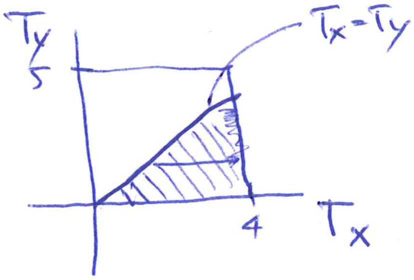
$$= 1578.83$$

Question No. 9:

You are given the joint density of (T_x, T_y):

$$f_{T_xT_y}(s,t) = \frac{1}{90}(s+t), \quad \text{for } 0 < s < 4, \ 0 < t < 5.$$

Calculate the probability (x) outlives (y).



$$\Pr[T_x \geq T_y] = \int_0^4 \int_t^4 \frac{1}{90}(s+t)\,ds\,dt$$

$$= \int_0^4 \frac{1}{90}\left(\frac{1}{2}s^2 + ts\right)\Big|_t^4 \, dt$$

$$= \int_0^4 \frac{1}{90}\left(\frac{1}{2}16 + 4t - \frac{1}{2}t^2 - t^2\right) dt$$

$$= \frac{1}{90}\int_0^4 \left(8 + 4t - \frac{3}{2}t^2\right) dt$$

$$= \frac{1}{90}\left(8(4) + \frac{1}{2}4(4)^2 - \frac{\cancel{3}}{2}\frac{1}{\cancel{3}}(4)^3\right)$$

$$= \frac{1}{90}\left(\cancel{32} + 32 - \cancel{32}\right)$$

$$= \frac{32}{90} = \underline{\underline{0.3556}}$$

Question No. 10:

You are given:

- ${}_{10|}q_{\overline{50:60}} = 0.0105$
- ${}_{10}p_{50} = 0.800$
- ${}_{10}p_{60} = 0.750$
- $p_{60} = 0.975$

Calculate q_{70}.

$$
\begin{aligned}
{}_{10|}q_{\overline{50:60}} &= {}_{11}q_{\overline{50:60}} - {}_{10}q_{\overline{50:60}} \\
&= {}_{11}q_{50}\,{}_{11}q_{60} - {}_{10}q_{50}\,{}_{10}q_{60} \\
&= (1-{}_{11}p_{50})\,{}_{11}q_{60} - (1-{}_{10}p_{50})(1-{}_{10}p_{60}) \\
&= \underbrace{(1-.8(.975))}\,{}_{11}q_{60} - (1-.8)(1-.75) \\
&= .22\,{}_{11}q_{60} - .05 = .0105
\end{aligned}
$$

Solving for ${}_{11}q_{60}$, we get ${}_{11}q_{60} = (.0105+.05)/.22$
$= .275$

But ${}_{11}q_{60} = {}_{10}q_{60} + {}_{10}p_{60}\,q_{70}$

$= .25 + .75\,q_{70} = .275$

$$q_{70} = \frac{.275-.25}{.75} = \frac{.025}{.75} = \frac{1}{30}.$$

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