MATH 3631
Actuarial Mathematics II
Class Test 2 - 3:35-4:50 PM
Monday, 13 April 2020
Time Allowed: as verbally agreed
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: ______________________________ **Student ID:** ______________________

PLEASE READ AND FOLLOW THESE INSTRUCTIONS:

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide DETAILS of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Please keep in mind that while taking this test, you are expected to act in accordance to the statement of Academic Integrity of the University of Connecticut.**
- **Please submit your answers with a PDF file starting with your last name followed by an underscore and whatever else you wanna name it. For example, Valdez_ClassTest2.pdf**

Question No. 1:

A health insurance company classifies its policyholders according to three states:

(0) preferred

(1) standard

(2) substandard

Transitions are annual and follow a time-homogeneous Markov model. The one-year transition probabilities are:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \quad \end{array} \\ \begin{pmatrix} 0.80 & 0.15 & 0.05 \\ 0.50 & 0.40 & 0.10 \\ 0.01 & 0.25 & 0.74 \end{pmatrix} \end{array}$$

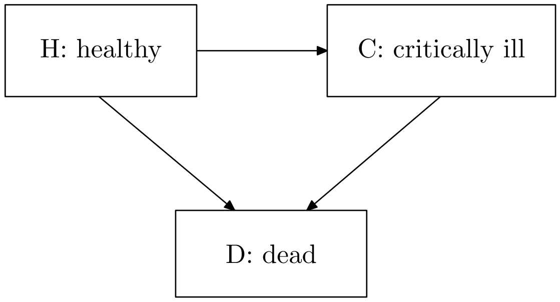
At the start of each year that the policyholder is classified 'standard', the premium is P. This premium is reduced by 25% if the customer moves to a 'preferred' state but is increased by 40% if the customer moves to a 'substandard' state.

Today, a customer purchases a policy and the company classified him as 'substandard'. The customer will keep its policy with this same company for the next 3 years, and the expected present value of his premiums is equal to 2,000 at an interest rate of $i = 0.05$.

Calculate P, the annual premium for a 'standard' policyholder.

Question No. 2:

A life insurer uses the following three-state model to price a two-year critical illness policy issued to healthy policyholders at time $t = 0$:



You are given:

- The constant forces of transition are:

$$\mu^{\text{HC}} = 0.010 \qquad \mu^{\text{HD}} = 0.005 \qquad \mu^{\text{CD}} = 0.025$$

- The policy pays 10,000 at the moment of death of the policyholder from the critically ill state. No other benefits are provided.

- $\delta = 5\%$

Calculate the actuarial present value of the benefits for this critical illness policy.

Question No. 3:

You are given the following extract from a triple-decrement table:

age x	number of survivors $\ell_x^{(\tau)}$	heart disease $q_x^{(1)}$	accidents $q_x^{(2)}$	other causes $q_x^{(3)}$
73	75,085	0.010	–	0.040
74	–	0.015	0.008	0.075
75	–	0.020	0.010	0.092
76	56,372	–	–	–

Calculate the number of deaths due to accidents between ages 73 and 74.

Question No. 4:

For a special 3-year endowment insurance on (55), you are given:

- The following two-decrement table, where decrement w refers to withdrawal and d refers to death:

x	$\ell_x^{(\tau)}$	$d_x^{(w)}$	$d_x^{(d)}$
55	10000	300	300
56	9400	150	400
57	8850	0	500

- The death benefit is 1000, payable at the end of the year of death.
- The endowment benefit is B, payable at the end of year 3 if the policyholder is alive.
- There are no benefits upon withdrawal.
- $i = 0.05$
- The actuarial present value for this insurance is 2,000.

Calculate B.

Question No. 5:

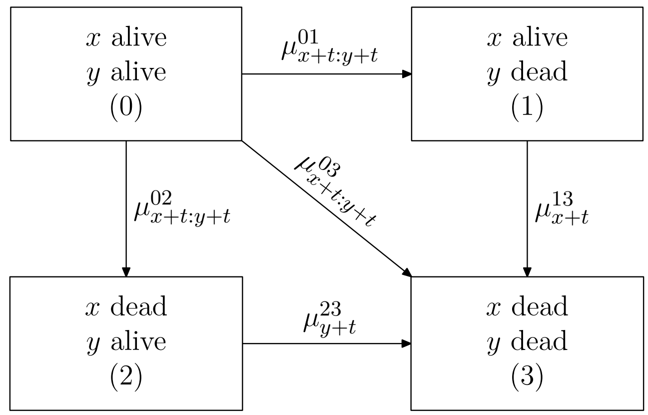
For two lives, (x) and (y), with independent future lifetimes, you are given:

k	q_{x+k}	q_{y+k}
0	0.10	0.08
1	0.15	0.08
2	0.20	0.10

Calculate ${}_3p_{\overline{xy}}$. Interpret this probability.

Question No. 6:

A special joint whole life insurance is issued on two lives, (x) and (y), whose joint mortality is a multiple state model with common shock:



You are given:

- The policy pays 2500 at the moment of simultaneous death, if that occurs, and zero otherwise.
- The following forces of intensities:

$$\mu^{01} = 0.010 \qquad \mu^{02} = 0.025 \qquad \mu^{03} = 0.005$$

- The actuarial present value of this insurance is 175.

Calculate the force of interest.

Question No. 7:

For two lives, (x) and (y), with independent future lifetimes, you are given:

- The probability that (x) dies after (y) and within one year is 0.063.
- The probability that (y) dies before (x) and within one year is 0.474.
- The probability that both (x) and (y) are dead within one year is 0.117.

Calculate q_x.

Question No. 8:

An insurance is issued on two lives, (x) and (y), with independent future lifetimes. You are given:

- The insurance pays 100 at the moment of death of (x), provided (x) dies after (y).
- Both lives are subject to the same mortality, each with a constant force of mortality equal to 0.05.
- $\delta = 0.05$

Calculate the actuarial present value for this insurance.

Question No. 9:

For two lives (50) and (60) with independent future lifetimes, you are given:

- ${}_{10}p_{50} = 0.95$
- ${}_{10}p_{\overline{50:60}} = 0.99$

Calculate ${}_{10}p_{50:60}$.

Question No. 10:

For an annuity immediate on a husband and wife of the same age 95, you are given:

- Their future lifetimes are independent.
- $q_x = \begin{cases} 0.8, & x = 95, 96, \ldots, 99 \\ 1.0, & x = 100 \end{cases}$
- $i = 0.10$
- The reversionary annuity pays 1 each year after the death of the husband.

Calculate the actuarial present value of this reversionary annuity.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK