MATH 3631
Actuarial Mathematics II
Class Test 2 - 3:35-4:50 PM
Monday, 13 April 2020
Time Allowed: as verbally agreed
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL **Student ID:** Suggested Solutions

PLEASE READ AND FOLLOW THESE INSTRUCTIONS:

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide DETAILS of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Please keep in mind that while taking this test, you are expected to act in accordance to the statement of Academic Integrity of the University of Connecticut.**
- **Please submit your answers with a PDF file starting with your last name followed by an underscore and whatever else you wanna name it. For example,** Valdez_ClassTest2.pdf

Question No. 1:

initial state vector = (0 0 1)

A health insurance company classifies its policyholders according to three states:

(0) preferred

(1) standard

(2) substandard

$(0\ 0\ 1) * Q = (.01\ \ .25\ \ .74)$

$(.01\ \ .25\ \ .74) * Q = (0.1404,\ 0.2865,\ 0.5731)$

Transitions are annual and follow a time-homogeneous Markov model. The one-year transition probabilities are:

$$Q \Leftarrow \begin{array}{cc} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0.80 & 0.15 & 0.05 \\ 0.50 & 0.40 & 0.10 \\ 0.01 & 0.25 & 0.74 \end{pmatrix} \end{array}$$

At the start of each year that the policyholder is classified 'standard', the premium is P. This premium is reduced by 25% if the customer moves to a 'preferred' state but is increased by 40% if the customer moves to a 'substandard' state.

Today, a customer purchases a policy and the company classified him as 'substandard'. The customer will keep its policy with this same company for the next 3 years, and the expected present value of his premiums is equal to 2,000 at an interest rate of $i = 0.05$.

$.75P$

P

$1.4P$

Calculate P, the annual premium for a 'standard' policyholder.

Probabilities * Cash Flows $\quad v = 1/1.05$

Year 0: $1.4P$

Year 1: $.75P(.01) + P(.25) + 1.4P(.74) = 1.2935P$

Year 2: $.75P(.1404) + P(.2865) + 1.4P(.5731) = 1.19414P$

$$\text{APV (premiums)} = 1.4P + 1.2935Pv + 1.19414Pv^2$$

$$= 3.715025P \Rightarrow 2{,}000$$

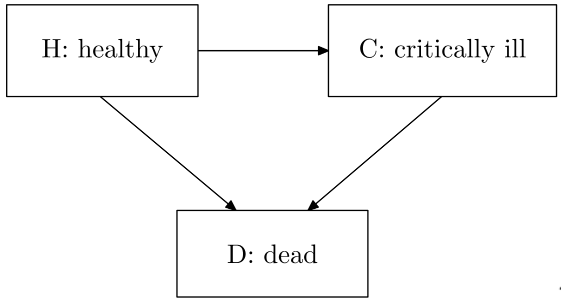
Solving for P, we get $P = \dfrac{2000}{3.715025} = 538.3544$

$\approx \underline{\underline{538.35}}$

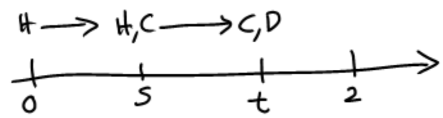
Question No. 2:

A life insurer uses the following three-state model to price a two-year critical illness policy issued to healthy policyholders at time $t = 0$:

at $t=0$



2 years only



You are given:

- The constant forces of transition are:

  $\mu^{HC} = 0.010 \qquad \mu^{HD} = 0.005 \qquad \mu^{CD} = 0.025$

- The policy pays 10,000 at the moment of death of the policyholder from the critically ill state. No other benefits are provided.
- $\delta = 5\%$

Calculate the actuarial present value of the benefits for this critical illness policy.

$$APV = \int_0^2 \int_0^t 10{,}000\, e^{-.05t}\, {}_sp^{HH}\, \mu^{HC}\, {}_{t-s}p^{CC}\, \mu^{CD}\, ds\, dt$$

$$= 10{,}000 * \int_0^2 \int_0^t e^{-.05t}\, e^{-.015s} (0.01)\, e^{-.025(t-s)}\, 0.025\, ds\, dt$$

$$= 10{,}000 * \cancel{.01} * 0.025 \underbrace{\int_0^2 \int_0^t e^{-.075t}\, e^{.01s}\, ds\, dt}_{\int_0^2 e^{-.075t} \cancel{\frac{1}{.01}}(e^{.01t}-1)\, dt}$$

$$= 10{,}000 * .025 \underbrace{\int_0^2 \left(e^{-.065t} - e^{-.075t}\right) dt}_{\frac{1}{.065}\left(1-e^{-.065(2)}\right) - \frac{1}{.075}\left(1-e^{-.075(2)}\right)}$$

$$= 4.556982 \approx \underline{\underline{4.56}}$$

Question No. 3:

You are given the following extract from a triple-decrement table:

age x	number of survivors $\ell_x^{(\tau)}$	heart disease $q_x^{(1)}$	accidents $q_x^{(2)}$	other causes $q_x^{(3)}$
73	75,085	0.010	–	0.040
74	–	0.015	0.008	0.075
75	–	0.020	0.010	0.092
76	56,372	–	–	–

Calculate the $\underbrace{\text{number of deaths due to accidents between ages 73 and 74}}_{d_{73}^{(2)}}$.

$$\frac{56{,}372}{\ell_{75}^{(\tau)}} = 1-(.020+.010+0.092) \Rightarrow \ell_{75}^{(\tau)} = \frac{56372}{0.878} = 64205.01$$

$$\frac{64205.01}{\ell_{74}^{(\tau)}} = 1-(.015+.008+.075) \Rightarrow \ell_{74}^{(\tau)} = \frac{64205.01}{0.902} = 71180.72$$

$$\frac{71180.2}{75085} = 1-(.010+q_{73}^{(2)}+.040) \Rightarrow q_{73}^{(2)} = \underbrace{0.95-\frac{71180.72}{75085}}$$

$$= 0.001998135$$

$$\approx 0.002$$

Therefore, $d_{73}^{(2)} = \underbrace{75085 * 0.002}_{150.17} \approx \underline{\underline{150}}$

Question No. 4:

For a special 3-year endowment insurance on (55), you are given:

- The following two-decrement table, where decrement w refers to withdrawal and d refers to death:

x	$\ell_x^{(\tau)}$	$d_x^{(w)}$	$d_x^{(d)}$
55	10000	300	300
56	9400	150	400
57	8850	0	500

- The death benefit is 1000, payable at the end of the year of death.
- The endowment benefit is B, payable at the end of year 3 if the policyholder is alive.
- There are no benefits upon withdrawal.
- $i = 0.05$

$V = 1/1.05$

- The actuarial present value for this insurance is 2,000. APV

Calculate B.

$$\text{APV(insurance)} = \underbrace{1\cancel{00}0\left(\frac{300}{10\cancel{000}}V + \frac{400}{10\cancel{000}}V^2 + \frac{500}{10\cancel{000}}V^3\right)} + B\,\frac{(\overbrace{8850-500}^{8350})}{10000}V^3$$

$$= 108.0445 + .835V^3 B \Rightarrow 2000$$

$$B = \frac{2000 - 108.0445}{.835V^3} = 2{,}622.964 \approx \underline{\underline{2{,}623}}$$

Question No. 5:

For two lives, (x) and (y), with independent future lifetimes, you are given:

k	q_{x+k}	q_{y+k}
0	0.10	0.08
1	0.15	0.08
2	0.20	0.10

Calculate ${}_3p_{\overline{xy}}$. Interpret this probability.

${}_3p_{\overline{xy}}$ = probability that in 3 years, at least one of (x) or (y) is still alive.

$$= 1 - {}_3q_{\overline{xy}} = 1 - {}_3q_x * {}_3q_y$$

$$= 1-(1-{}_3p_x)*(1-{}_3p_y)$$

$$= 1-(1-.9*.85*.80)(1-.92*.92*.9)$$

$$= 0.9075629 \approx \underline{\underline{0.9076}}$$

Question No. 6:

A special joint whole life insurance is issued on two lives, (x) and (y), whose joint mortality is a multiple state model with common shock:

$$
\begin{tikzcd}
\boxed{\begin{matrix} x \text{ alive} \\ y \text{ alive} \\ (0) \end{matrix}} \arrow[r, "\mu^{01}_{x+t:y+t}\ (.01)"] \arrow[d, "\mu^{02}_{x+t:y+t}", "(.025)"'] \arrow[dr, "\mu^{03}_{x+t:y+t}\ (.005)"] & \boxed{\begin{matrix} x \text{ alive} \\ y \text{ dead} \\ (1) \end{matrix}} \arrow[d, "\mu^{13}_{x+t}"] \\
\boxed{\begin{matrix} x \text{ dead} \\ y \text{ alive} \\ (2) \end{matrix}} \arrow[r, "\mu^{23}_{y+t}"] & \boxed{\begin{matrix} x \text{ dead} \\ y \text{ dead} \\ (3) \end{matrix}}
\end{tikzcd}
$$

1

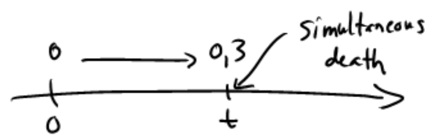
You are given:

- The policy pays 2500 at the moment of simultaneous death, if that occurs, and zero otherwise.
- The following forces of intensities:

$$\mu^{01} = 0.010 \qquad \mu^{02} = 0.025 \qquad \mu^{03} = 0.005$$

- The actuarial present value of this insurance is 175.

Calculate the force of interest.



$$APV = \int_0^\infty 2500\, e^{-\delta t}\, e^{-.04t} * .005 \; dt$$

$$= 2500(.005) \underbrace{\int_0^\infty e^{-(.04+\delta)t}\, dt}_{\frac{1}{.04+\delta}} = 175$$

Solving for δ, we get

$$\delta = \frac{2500(.005)}{175} - .04 = .0314857 \approx .03 \qquad 3\%$$

Question No. 7:

For two lives, (x) and (y), with independent future lifetimes, you are given:

- The probability that (x) dies after (y) and within one year is 0.063. $\rightarrow$ $q^{2}_{xy} = .063$
- The probability that (y) dies before (x) and within one year is 0.474. $\rightarrow$ $q_{x\overset{1}{y}} = .474$
- The probability that both $\underbrace{(x) \text{ and } (y)}_{\text{last death}}$ are dead within one year is 0.117. $\rightarrow$ $q_{\overline{xy}} = .117$

Calculate q_x.

$$q_{\overline{xy}} = \underbrace{q^{2}_{xy}}_{.063} + q_{x\overset{2}{y}}$$

$$\Rightarrow q_{x\overset{2}{y}} = .117 - .063 = .054$$

$$q_{\overline{xy}} = q_x \times q_y$$

$$q_{x\overset{1}{y}} + q_{x\overset{2}{y}} = q_y$$

$$.474 + .054 = 0.528$$

$$\frac{.117}{\underbrace{q_y}_{.528}} = q_x \Rightarrow q_x = 0.2215909$$

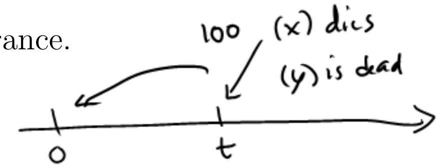
$$\approx 0.222$$

Question No. 8:

An insurance is issued on two lives, (x) and (y), with independent future lifetimes. You are given:

- The insurance pays 100 at the moment of death of (x), provided (x) dies after (y).
- Both lives are subject to the same mortality, each with a constant force of mortality equal to 0.05.
- $\delta = 0.05$

Calculate the actuarial present value for this insurance.



$$\text{APV (insurance)} = \int_0^\infty 100\, e^{-.05t}\, \underbrace{{}_tp_x}_{e^{-.05t}}\, \underbrace{\mu_{x+t}}_{.05} \cdot \underbrace{{}_tq_y}_{(1-e^{-.05t})}\, dt$$

$$= 100 * .05 \left(\int_0^\infty e^{-.10t}\, dt - \int_0^\infty e^{-.15t}\, dt \right)$$

$$= \underbrace{100 * .05 \left(\frac{1}{.10} - \frac{1}{.15} \right)}$$

$$= \underline{\underline{16.67}}$$

Question No. 9:

For two lives (50) and (60) with independent future lifetimes, you are given:

- ${}_{10}p_{50} = 0.95$
- ${}_{10}p_{\overline{50:60}} = 0.99$ $\longrightarrow$ $1 - {}_{10}q_{\overline{50:60}} = 1 - \underbrace{{}_{10}q_{50}}_{.05} * {}_{10}q_{60} = .99$

$\Rightarrow \quad {}_{10}q_{60} = \dfrac{.01}{.05} = .20 \Rightarrow {}_{10}p_{60} = .80$

Calculate ${}_{10}p_{50:60}$.

$= {}_{10}p_{50} * {}_{10}p_{60}$

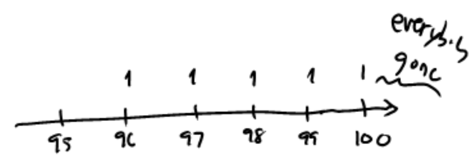
$= 0.95 * 0.80 = \underline{\underline{0.76}}$

Question No. 10:

For an annuity immediate on a husband and wife of the same age 95, you are given:

- Their future lifetimes are independent.
- $q_x = \begin{cases} 0.8, & x = 95, 96, \ldots, 99 \\ 1.0, & x = 100 \end{cases}$
- $i = 0.10$
- The reversionary annuity pays 1 each year after the death of the husband.

Calculate the actuarial present value of this reversionary annuity. $v = 1/1.10$



$$\text{APV(reversionary annuity)} = a_{\underset{\text{husband}}{95}|\underset{\text{wife}}{95}} = a_{95} - a_{95:95}$$

$$a_{95} = \sum_{k=1}^{\infty} v^k \, {}_kp_{95} = \sum_{k=1}^{5} \left(\frac{1}{1.10}\right)^k (.20)^k$$

$$= \frac{.2}{1.1}\left(\frac{1-(.20/1.10)^5}{1-.20/1.10}\right) = .2221781$$

$.2^2 = .04$

$$a_{95:95} = \sum_{k=1}^{5} v^k \, {}_kp_{95} \, {}_kp_{95}$$

$$= \sum_{k=1}^{5} \left(\frac{(.20)^2}{1.10}\right)^k$$

$$= \frac{.04}{1.10}\left(\frac{1-(.04/1.10)^5}{1-(.04/1.10)}\right) = .03773585$$

$$a_{95|95} = a_{95} - a_{95:95}$$

$$= 0.1844422 \approx \underline{\underline{0.184}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK