MATH 3631
Actuarial Mathematics II
Class Test 2 - 5:00-6:15 PM
Wednesday, 4 April 2018
Time Allowed: 1 hour and 15 minutes
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

ABC Life Insurance Company issues fully discrete 20-year term insurance policies of 1000 to (45). You are given:

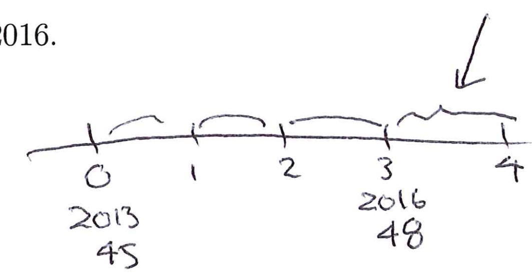
- Expected mortality are: $q_{48} = 0.005$ $q_{49} = 0.006$ $q_{50} = 0.007$
- Per policy expenses, incurred at the beginning of the year, consist of 1.5 in the first year and 0.5 thereafter.
- Death-related expense of 120 is payable at the end of the year of death.
- ${}_4V$, the net premium reserve at the end of year 4 per policy, is 16.05.

On 1 January 2013, ABC sold 15,000 of these policies to lives all aged 45. You are now also given that:

- During the first three years, there were 315 actual deaths from these policies.
- During 2016, there were 19 actual deaths from these policies.
- Gains or losses are calculated in the following order: interest $\to$ mortality $\to$ expenses.

Calculate ABC's gain or loss due to mortality for year 2016.

$N_3 = 15{,}000 - 315 = 14{,}685$



gain/loss from mortality

$$= (\underbrace{1000}_{B} + \underbrace{120}_{\text{death expense}} - \underbrace{16.05}_{{}_4V}) \times (\underbrace{.005(14{,}685)}_{\text{expected \# of deaths}} - \underbrace{19}_{\text{actual \# of deaths}})$$

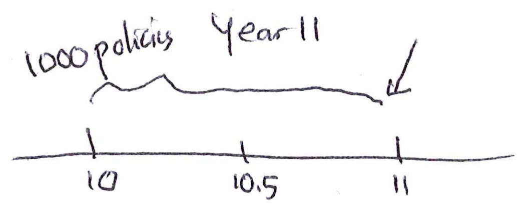
$$= +60{,}082.48$$

Question No. 2:

For a whole life insurance of 10,000 with semi-annual premiums on (65), you are given:

- A gross premium of 90 is payable every 6 months starting at age 65.
- Actual expenses incurred are 10% for each time a premium is paid.
- Death benefits are paid at the end of the half year of death.
- During year 11, interest earned is $i^{(2)} = 0.08$.
- There were remaining 1000 policies at the beginning of year 11.
- During year 11, there were 25 deaths in the first half and another 30 deaths for the rest of the year.
- The asset share at the end of year 10 is 3,892.50.

Calculate the asset share at the end of year 11.



$$AS_{10.5} = \frac{(3892.50 + 90\times(1-.1))(1.04) - 10000 \times 25/1000}{1 - 25/1000}$$

$$= 3981.99$$

$$AS_{11} = \frac{(3981.99 + 90\times(1-.1))(1.04) - 10000 \times 30/975}{1 - 30/975}$$

$$= \underline{\underline{4,042.192}}$$

Question No. 3:

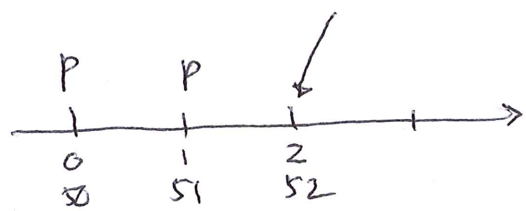
Toby, age 50, buys a fully discrete whole life insurance policy with a death benefit of 500,000. Immediately before the third annual premium payment is due, he stops paying premiums and the policy is converted to a paid-up policy with a reduced death benefit.

Original and conversion pricing were based on the following:

- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$
- The equivalence principle.

The cash value of the policy is equal to 90% of the net premium reserve.

Calculate the death benefit of the reduced paid-up policy.



Let P be the net annual premium

$$P = 500000 \frac{A_{50}}{\ddot{a}_{50}} = 9386.212$$

A_{50} .24905

$\ddot{a}_{50}$ 13.2668

$${}_0V = 0$$

$${}_1V = \frac{P(1+i) - 500000\, q_{50}}{1 - q_{50}} = 7031.009$$

q_{50} 5.92/1000

$${}_2V = \frac{({}_1V + P)(1+i) - 500000\, q_{51}}{1 - q_{51}} = 14283.960$$

q_{51} 6.42/1000

$$CV_2 = 0.90 \times {}_2V = 12855.56$$

Let RPU be the reduced paid up benefit

$$RPU = CV_2 / A_{52} = \frac{12855.56}{.27050} = \underline{\underline{47,525.18}}$$

Question No. 4:

For a fully discrete whole life insurance of 100,000 on (45), you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$
- Commission expenses are 60% of the first year gross premium and 2% of renewal gross premiums.
- Administrative expenses are 500 in the first year and 50 in each subsequent year.
- All expenses are payable at the beginning of the year.
- The gross premium, calculated using the equivalence principle, is 1605.72.

Calculate the expense reserve at the end of year 10.

Net annual premium: $P = 100000\, \underbrace{A_{45}}_{.20120} / \underbrace{\ddot{a}_{45}}_{14.1121} = 1425.727$

expense loading: $el = G - P = 1605.75 - 1425.727 = 179.993$

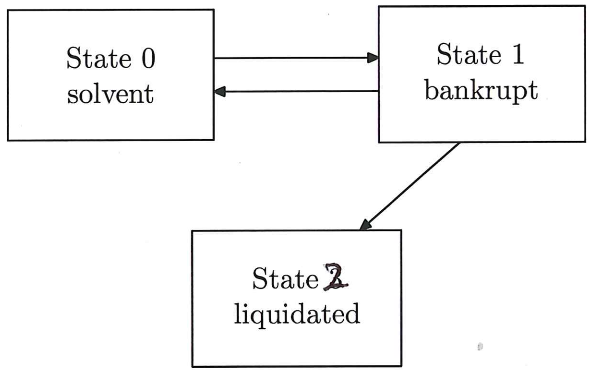
$$_{10}V^e = APV(FE_{10}) - APV(Fel_{10})$$

$$= (.02\underbrace{G}_{1605.75} + 50)\,\ddot{a}_{55} - 179.993\,\underbrace{\ddot{a}_{55}}_{12.2758}$$

$$= -1{,}201.538$$

Question No. 5:

The financial strength of a company is based on the following Markov model:



You are given the constant forces of transition:

$$\mu^{01} = 0.01 \qquad \mu^{10} = 0.05 \qquad \mu^{12} = 0.10$$

In calculating transition probabilities, use the Kolmogorov's forward equations with the Euler approximation based on steps of $h = 1/2$.

Calculate the probability that a company currently *bankrupt* will be *solvent* at the end of one year.

$\rightarrow 1$ $\rightarrow 0$

at $t=0$, ${}_0p^{10} = {}_0p^{12} = 0$, ${}_0p^{11} = 1$

For $t=0$, $h=1/2$:

$${}_{1/2}p^{10} = \cancel{{}_0p^{10}} + 0.5\left[\left({}_0p^{11}\mu^{10} + \cancel{{}_0p^{12}\mu^{20}}\right) - \cancel{{}_0p^{10}}\left(\mu^{01}+\mu^{02}\right)\right]$$

$$= 0.5(1)(.05) = .025$$

$${}_{1/2}p^{12} = \cancel{{}_0p^{12}} + 0.5\left[\left(\cancel{{}_0p^{10}\mu^{02}} + {}_0p^{11}\mu^{12}\right) - \cancel{{}_0p^{12}\underbrace{\left(\mu^{20}+\mu^{21}\right)}_{\text{not possible}}}\right]$$

$$= 0.5(1)(.10) = .050$$

$${}_{1/2}p^{11} = 1 - {}_{1/2}p^{10} - {}_{1/2}p^{12} = 1 - .025 - .050 = 0.925$$

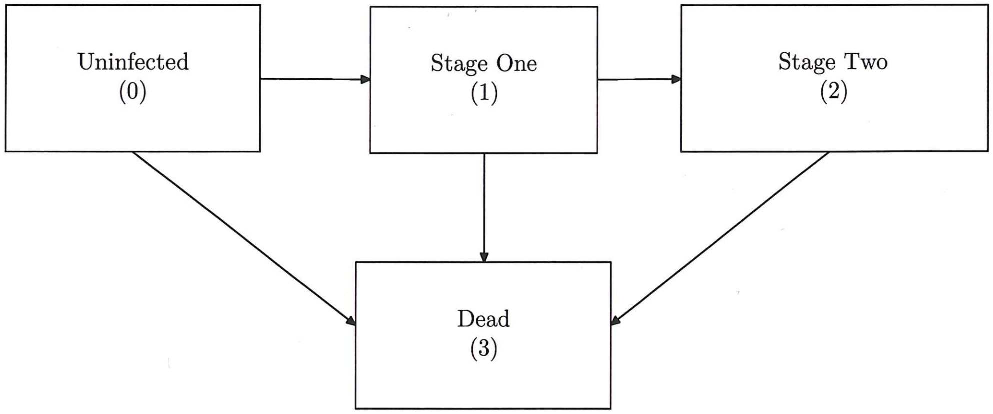
For $t=0.5$, $h=1/2$:

$${}_1p^{10} = {}_{1/2}p^{10} + 0.5\left[\left({}_{1/2}p^{11}\mu^{10} + \cancel{{}_{1/2}p^{12}\mu^{20}}\right) - {}_{1/2}p^{10}\left(\mu^{01}+\cancel{\mu^{02}}\right)\right]$$

$$= .025 + 0.5\left[.925(.05) - .025(.01)\right] = \underline{\underline{0.048}}$$

Question No. 6:

A disease progresses according to the following multiple state model:



An insurance company provides coverage for this disease by paying a fixed benefit amount of 100 at the moment an uninfected policyholder reaches 'Stage One' and twice that amount he/she reaches 'Stage Two'. You are given:

- All transition intensities are constant and independent of age:

$$\mu^{01} = 0.005, \quad \mu^{12} = 0.08, \quad \mu^{03} = 0.01, \quad \mu^{13} = 0.05, \quad \text{and} \quad \mu^{23} = 0.40$$

- $\delta = 0.05$

Calculate the actuarial present value for this insurance benefit.

$0 \longrightarrow 0,1$

$0 \qquad t$

$$APV(0 \to 1) = \int_0^\infty 100\, e^{-.05t}\, e^{-.015t} (.005)\, dt$$

$$= 100(.005) \int_0^\infty e^{-.065t}\, dt = 100 \frac{.005}{.065} = 7.692308$$

$0 \longrightarrow 0,1 \longrightarrow 1,2$

$0 \qquad s \qquad t$

$$APV(0 \to 2) = \int_0^\infty 200\, e^{-.05t} \int_0^t e^{-.015s} (.005)\, e^{-0.13(t-s)} (.08)\, ds\, dt$$

$$= 200(.005)(.08) \int_0^\infty e^{-.18t} \int_0^t e^{0.115s}\, ds\, dt$$

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continued:

$$= 200 \frac{(.005)(.08)}{.115} \underbrace{\int_0^\infty e^{-.18t}\left(e^{0.115t} - 1\right) dt}_{\left(\frac{1}{.065} - \frac{1}{0.18}\right)}$$

$$= 6.837607$$

$$APV = APV(0 \to 1) + APV(0 \to 2)$$

$$= 7.692308 + 6.837607$$

$$= \underline{\underline{14.52991}}$$

Question No. 7:

An actuarial student uses one of two manuals (Manual W and Manual Z) to prepare for an actuarial exam. You are given the following states:

(a) uses Manual W, but fails,

(b) uses Manual W, and passes,

(c) uses Manual Z, but fails, and

(d) uses Manual Z, and passes

You are given the annual transition probability matrix for a time-homogeneous Markov Chain:

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{pmatrix} 0.3 & 0.1 & 0.5 & 0.1 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix} \end{array}.$$

The actuarial exam is given once a year. It has been observed that generally 40% of the students uses Manual W while the rest uses Manual Z.

Calculate the probability that a student will pass only on the third attempt.

$Pr[\text{3rd attempt} \mid W] = 0.3(0.1) = 0.03$

$Pr[\text{3rd attempt} \mid Z] = 0.2(0.1) = 0.02$

$Pr[\text{3rd attempt}] = .03(0.40) + .02(0.60)$

$= \underline{\underline{0.024}}$

Question No. 8:

In a triple decrement table, you are given that decrement (1) is death, decrement (2) is disability, and decrement (3) is withdrawal. You are given:

- $q_{65}^{\prime(1)} = 0.05$, $q_{65}^{\prime(2)} = 0.10$ and $q_{65}^{\prime(3)} = 0.15$.
- Mortality is uniformly distributed over each year of age in its associated single decrement table.
- Disability occurs only in the middle of the year.
- Withdrawals occur only at the end of the year.

Calculate $q_{65}^{(1)}$.

$$\begin{cases} 1, & 0 \le t < 1/2 \\ 1 - q_{65}^{\prime(2)}, & 1/2 \le t < 1 \end{cases}$$

$$q_{65}^{(1)} = \int_0^1 \underbrace{{}_tp_{65}^{\prime(1)}\, \mu_{65+t}^{\prime(1)}}_{q_{65}^{\prime(1)}}\, {}_tp_{65}^{\prime(2)}\, \underbrace{{}_tp_{65}^{\prime(3)}}_{=1,\ 0 \le t < 1}\, dt$$

$$= \int_0^{1/2} .05\, dt + \int_{1/2}^1 .05(1-0.10)\, dt$$

$$= .05(1/2) + .05(.9)(1 - 1/2)$$

$$= \underline{\underline{0.0475}} \le q_{65}^{\prime(1)} = .05$$

check!

Question No. 9:

You are given:

- The following extract from a triple-decrement table:

x	$\ell_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
55	100,000	500	8,050	1,100
56	90,350	–	–	1,200
57	80,000	–	–	–

- All decrements are uniformly distributed over each year of age in the triple decrement table.
- $q_x'^{(2)}$ is the same for all ages.

Calculate $q_{56}'^{(1)}$.

$q_{55}^{(\tau)} = .0965 \quad q_{55}^{(2)} = .0805$

$q_{55}'^{(2)} = 1-(1-q_{55}^{(\tau)})^{q_{55}^{(2)}/q_{55}^{(\tau)}} = 1-(1-.0965)^{.0805/.0965} = .08116949$

also $= q_{56}'^{(2)}$

$q_{56}'^{(2)} = 1-(1-q_{56}^{(\tau)})^{q_{56}^{(2)}/q_{56}^{(\tau)}} \qquad q_{56}^{(\tau)} = 1-p_{56}^{(\tau)} = 1-80000/90350 = .1145545$

$\Rightarrow q_{56}^{(2)} = q_{56}^{(\tau)} \dfrac{\log(1-q_{56}'^{(2)})}{\log(1-q_{56}^{(\tau)})} = .07970659$

$q_{56}^{(1)} = q_{56}^{(\tau)} - q_{56}^{(2)} - q_{56}^{(3)} = .02156624$

$\Rightarrow q_{56}'^{(1)} = 1-(1-.1145545)^{.02156624/.1145545}$

$= \underline{\underline{.02264444}} \geq q_{56}^{(1)}$ check!

Question No. 10:

You are given the following extract from a triple-decrement table:

age x	no. of lives $\ell_x^{(\tau)}$	heart disease $q_x^{(1)}$	accidents $q_x^{(2)}$	other causes $q_x^{(3)}$
50	100,000	0.0009	0.0015	0.0010
51	–	0.0012	0.0020	0.0014
52	–	0.0014	0.0025	0.0018

Calculate ${}_3q_{50}^{(\tau)}$.

$$\ell_{51}^{(\tau)} = 100000 * (1-(.0009 + .0015 + .0010))$$

$$= \cancel{98850} \quad 99660$$

$$\ell_{52}^{(\tau)} = 98850 * (1-(.0012 + .0020 + .0014))$$

$$= \cancel{98395.29} \quad 99201.56$$

$$\ell_{53}^{(\tau)} = 98395.29 * (1-(.0014 + .0025 + .0018))$$

$$= \cancel{97834.44} \quad 98636.12$$

$${}_3q_{50}^{(\tau)} = 1 - {}_3p_{50}^{(\tau)} = 1 - \frac{\ell_{53}^{(\tau)}}{\ell_{50}^{(\tau)}} = 1 - \frac{\cancel{97834.44}\ 98636.12}{100000}$$

$$= \cancel{0.02165563} \quad .0136389$$

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