MATH 3631

Actuarial Mathematics II

Class Test 2 - 5:00-6:15 PM

Wednesday, 12 April 2017

Time Allowed: 1 hour

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

A fully discrete whole life insurance policy was issued 20 years ago to (30) with a death benefit equal to B. You are given:

- The policyholder decides to convert the policy to a paid-up whole life insurance policy with a paid-up benefit amount of 75,125.
- The policy's cash surrender value is equal to its net premium reserve.
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$

Calculate B.

13.2668

The policy's CSV is $_{20}V = B * \left(1 - \frac{\ddot{a}_{50}}{\ddot{a}_{30}}\right)$

15.8561

$= B * 0.1632999$

To be used for a paid-up policy:

0.24905

$B * 0.1632999 = 75{,}125 * A_{50}$

Solving for B, we get

$$B = \frac{75{,}125}{0.1632999}(0.24905)$$

$$= \underline{\underline{114{,}573.70}}$$

Question No. 2:

For a fully continuous whole life insurance policy issued to (40), you are given:

- The death benefit is 100,000.
- The annual gross premium is 1,790.
- Expenses are payable at the rate of 5% of the gross premium per year.
- $\delta = 0.03$
- The gross premium reserve at the end of 10 years is 17,500.
- $\mu_{40} = 0.001$ and $\mu_{50} = 0.005$

Use the Thiele's differential equation to calculate $\frac{d}{dt}\,{}_tV$ at $t = 10$.

Thiele's : $d\,{}_tV = \delta\,{}_tV + G_t - e_t - (B_t - {}_tV)\mu_{x+t}\,dt$

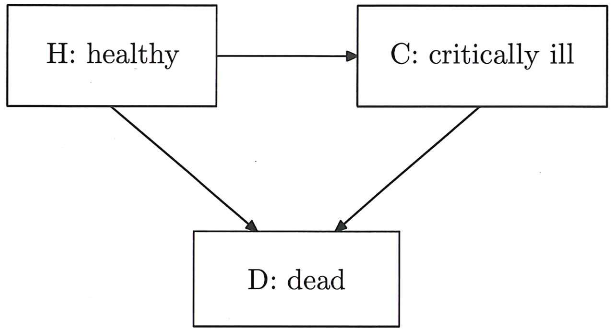
Thus, we set

$$\frac{d\,{}_tV}{dt} \text{ at } t=10 = .03(17500) + 1790 - .05(1790) - (100000 - 17500)(.005)$$

$$\simeq \underline{\underline{1813}}$$

Question No. 3:

A critical illness model is depicted according to the following multiple state model:



You are given the constant forces of transition:

$$\mu^{HC} = 0.03 \qquad \mu^{HD} = 0.02 \qquad \mu^{CD} = 0.05$$

For a healthy policyholder, denote by pH_{10} the probability that he will remain healthy by the end of 10 years, and by pC_{10} the probability that he will be critically ill by the end of 10 years.

Calculate $\dfrac{pH_{10}}{pC_{10}}$.

$$pH_{10} = e^{-\int_0^{10} (.03+.02)\,dt} = e^{-.5}$$

$H \to H,C \to C$

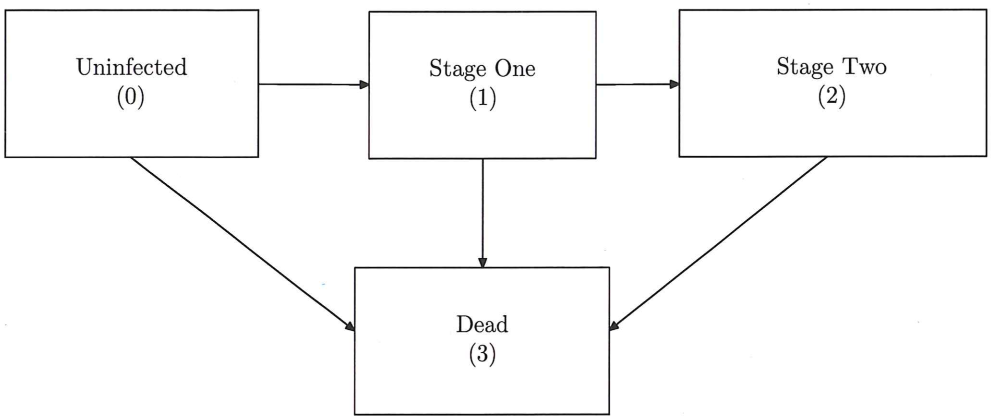
$0 \quad t \quad 10$

$$pC_{10} = \int_0^{10} \cancel{e^{-.05t}}\, 0.03\, e^{-.05(10-\cancel{t})}\,dt = .03\, e^{-.5}(10) = 0.3e^{-.5}$$

$$\frac{pH_{10}}{pC_{10}} = \frac{e^{-.5}}{.03e^{-.5}} = \frac{1}{.\cancel{0}3} = \frac{10}{3} = 3.3333$$

Question No. 4:

A disease progresses according to the following multiple state model:



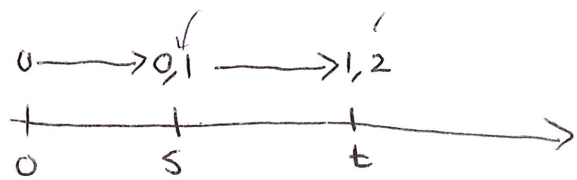
An insurance company provides coverage for this disease by paying a fixed benefit amount of 15,000 at the moment an uninfected policyholder reaches 'Stage Two'. You are given:

- All transition intensities are constant and independent of age:

$$\mu^{01} = 0.001, \quad \mu^{12} = 0.100, \quad \mu^{03} = 0.002, \quad \mu^{13} = 0.050, \quad \text{and} \quad \mu^{23} = 0.400$$

- $\delta = 0.05$

Calculate the actuarial present value for this insurance benefit.



$$\text{APV} = 15{,}000 * \int_0^\infty \int_0^t e^{-.05t} \cdot e^{-.003s} (.001)\, e^{-.15(t-s)} (0.10)\, ds\, dt$$

$$= 15{,}000(.0001) \int_0^\infty e^{-.20t} \int_0^t e^{.147s}\, ds\, dt$$

$$= \frac{15000(.0001)}{(.147)} \underbrace{\int_0^\infty e^{-.20t} \left(e^{.147t} - 1 \right) dt}_{\left(\frac{1}{.053} - \frac{1}{.20} \right)}$$

$$= \underline{141.5094}$$

Question No. 5:

An actuarial student uses one of two manuals (Manual W and Manual Z) to prepare for an actuarial exam. You are given the following states:

- (a) uses Manual W, but fails,
- (b) uses Manual W, and passes,
- (c) uses Manual Z, but fails, and
- (d) uses Manual Z, and passes

You are give the transition probability matrix for a time-homogeneous Markov Chain:

$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{pmatrix} 0.3 & 0.1 & 0.5 & 0.1 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix} \end{array}.$$

The actuarial exam is given once a year. Ten students, on their first attempt, all failed the actuarial exam using Manual Z.

Calculate the probability that exactly half of these 10 students will pass the actuarial exam only after two more attempts.

initial vector

$(0\ 0\ 1\ 0) * Q = (.5\ \ .2\ \ .2\ \ .1)$

$(.5\ \ .2\ \ .2\ \ .1) * Q = (.25\ \ .29\ \ .29\ \ .17)$

pass pass

prob of passing $= .29 + .17 = 0.46$ - deduct .3 prob of passing the first attempt

prob exactly 5 passes $= \binom{10}{5}(.16)^5(.84)^5$ $= .01105088$

$= \cancel{0.238318909}$

Question No. 6:

You are given the following extract from a triple-decrement table:

age x	no. of lives $\ell_x^{(\tau)}$	heart disease $q_x^{(1)}$	accidents $q_x^{(2)}$	other causes $q_x^{(3)}$
50	100,000	0.0009	0.0015	0.0010
51	–	0.0012	0.0020	0.0014
52	–	0.0014	0.0025	0.0018

Calculate $d_{52}^{(3)}$.

$$\ell_{51}^{(\tau)} = 100000(1 - .0009 - .0015 - .0010)$$

$$= 99{,}660$$

$$\ell_{52}^{(\tau)} = 99660(1 - .0012 - .0020 - .0014)$$

$$= 99{,}201.56$$

$$d_{52}^{(3)} = .0018\,(99201.56)$$

$$= \underline{\underline{178.5628}}$$

Question No. 7:

In a triple decrement table, you are given that decrement (1) is death, decrement (2) is disability, and decrement (3) is withdrawal. You are given:

- $q_{65}^{\prime(1)} = 0.02$, $q_{65}^{\prime(2)} = 0.08$ and $q_{65}^{\prime(3)} = 0.20$.
- Withdrawals occur only at the end of the year.
- Mortality and disability are uniformly distributed over each year of age in the associated single decrement tables.

Calculate $q_{65}^{(2)}$.

$$\text{UDD} \Rightarrow q_{65}^{\prime(2)} \qquad \mu_{65+t}^{\prime(2)}$$

$$q_{65}^{(2)} = \int_0^1 {}_tp_{65}^{\prime(1)}\ {}_tp_{65}^{\prime(2)}\ \underbrace{{}_tp_{65}^{\prime(3)}}_{=1,\ 0\le t<1}\ dt \qquad \text{eoy}$$

$$= q_{65}^{\prime(2)} \int_0^1 (1 - t \cdot q_{65}^{\prime(1)})\, dt$$

$$= .08 \int_0^1 (1 - .02t)\, dt$$

$$= .08\left(1 - \frac{.02}{2}\right) = \underline{\underline{.0792}} < \underline{.08}$$

Question No. 8:

Ehman, now exactly age 59, is a newly-hired actuary at Get-a-Life insurance company. His contractual employment is only for the next two years but he receives as fringe benefits the following:

- a death benefit of 10,000 at the end of the year of death while employed;
- a one-time disability payment of 5,000 at the end of the year of disability while employed;
- nothing if he withdraws or retires while employed; and
- a benefit of 25,000 at the end of his contractual period if he is still alive.

Decrements, while employed, follow the `Illustrative Service Table`, which is attached. Interest rate $i = 0\%$.

Calculate the total actuarial present value of Ehman's fringe benefits.

Since $i = 0\%$, $v = 1$ in all the APV calculations

$$\text{APV(death benefits)} = 10000\left(\frac{316}{24505} + \frac{313}{24505}\right) = 256.6823$$

$$\text{APV(disability benefits)} = 5000\left(\frac{213}{24505}\right) = 43.46052$$

$$\text{APV(survival benefits)} = 25000\left(\frac{19991}{24505}\right) = 20394.82$$

add all APV's, we get

$$\underbrace{256.6823 + 43.46052 + 20394.82}$$

$$= \underline{\underline{20,694.96}}$$

Question No. 9:

For a Type B Universal Life policy with additional death benefit of 5,000 issued to (50), you are given:

- Expense charges in each year are 1.5% of premium plus 20.
- The cost of insurance rate is equal to 125% of the mortality rate at the attained age based on the `Illustrative Life Table`.
- $i^c = 6\%$ for all years
- $i^q = 5\%$ for all years
- The account value at the end of the first year is equal to 1,194.37.
- The corridor factor requirement is a minimum of 2.0 each year.

Calculate the largest amount of premium this policyholder can pay at the beginning of the second year.

$$\frac{6.42}{1000}$$

$$AV_2 = \underbrace{\left[1194.37 + \pi_1(1-.015) - 20 - 1.25\, q_{51} \frac{1}{1.05} 5000\right] \times 1.06}_{\leq 5000}$$

$$CF = \frac{AV_2 + 5000}{AV_2} \geq 2.0 \implies AV_2 \leq 5000$$

Solve for π_1, we get

$$\pi_1 \leq \underbrace{\frac{5000 - \left(1194.37 - 20 - 1.25 \frac{6.42}{1000} \frac{1}{1.05} 5000\right)(1.06)}{(1-.015)(1.06)}}$$

$$\underline{\underline{3635.356}}$$

Question No. 10:

Tom, now exactly age 64, buys a Type A Universal Life policy with a total death benefit of 100,000. You are given:

policy year	annual premium deposit	percent of premium charge	annual fixed expense charge	annual cost of insurance rate per 1,000	interest credited
1	25,000	10%	200	3.0	5%

Tom decided to surrender his UL policy at the end of the first year and uses the entire balance of his policy's account value to purchase a life annuity-immediate that pays P each year while alive. This annuity will be calculated based on:

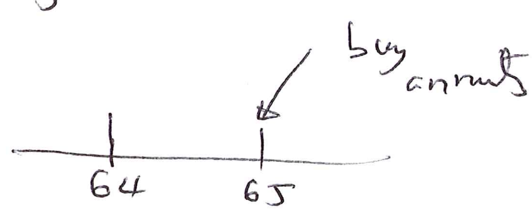
- Mortality follows the `Illustrative Life Table`.
- $i = 0.06$.

Calculate P.

$$AV_1 = \left(25000(1-.10) - 200\right)(1.05) - (100000 - AV_1) * \frac{3}{1000}$$

$$AV_1 = \frac{23115}{1 - 3/1000} = 23,184.55$$

all to be used to buy life annuity-immediate



$$23184.55 = P\, a_{65}$$

$$\ddot{a}_{65} - 1 = 8.8969$$

$$P = \frac{23184.55}{8.8969} = \underline{\underline{2,605.913}}$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK