MATH 3631
Actuarial Mathematics II
Class Test 2 - 3:35-4:50 PM
Wednesday, 20 April 2016
Time Allowed: 1 hour
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____ Student ID: _____

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

An insurance company issued 500,000 fully discrete whole life insurance policies to lives all exactly age 40 on January 1, 2002. Each policy issued has a death benefit of \$50,000 with an annual gross premium of \$375.

You are given:

- The following values in Year 2012:

	anticipated	actual
Expenses as a percent of premium	0.05	0.04
Fixed expenses	7	10
Death settlement expense	100	150
Annual effective rate of interest	0.050	0.063
q_{50}	0.0012	-

- The gross premium reserves per policy at the end of Years 2011 and 2012 are, respectively:

$$ {}_{10}V^g = 3,539 \quad \text{and} \quad {}_{11}V^g = 4,027 $$

- A total of 495,100 remain in force at the beginning of Year 2012.
- Gains and losses are calculated in the following order: expenses then mortality then interest.
- There is a total of zero gain or loss for Year 2012.

Calculate the gain or loss due to mortality in Year 2012.

Question No. 2:

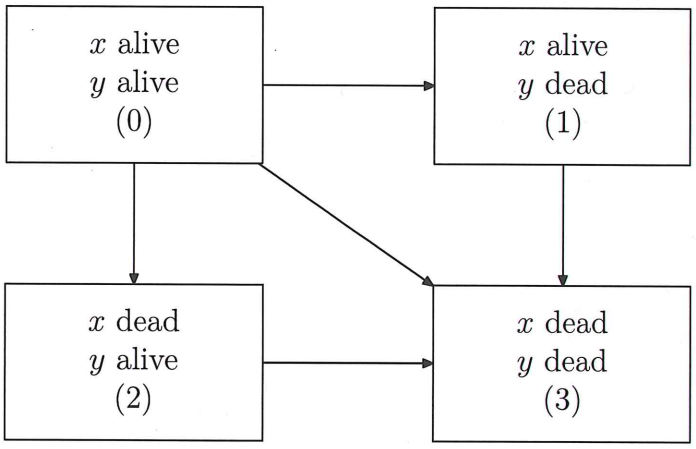
For a fully discrete whole life insurance policy of \$150,000 issued to age 40, you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 6\%$

Calculate the Full Preliminary Term (FPT) reserve at the end of 15 years.

Question No. 3:

The joint lifetime of (x) and (y) is being modeled as:



Assume that at time 0, both (x) and (y) are alive. All transition intensities are constant and independent of age:

$$\mu^{01} = 0.035, \quad \mu^{02} = 0.015, \quad \mu^{03} = 0.001, \quad \mu^{13} = 0.075, \quad \text{and} \quad \mu^{23} = 0.050$$

Calculate the probability that the first death is (x) and that the death of (y) occurs within the subsequent 10 years.

Question No. 4:

An insurer issues a special 2-year insurance to a high risk individual. You are given the following time-homogeneous Markov Chain model:

- The possible states are active (a), disabled (i), withdrawn (w) or dead (d) with annual transition matrix:

$$\begin{array}{c} \\ \text{a} \\ \text{i} \\ \text{w} \\ \text{d} \end{array} \begin{array}{c} \begin{array}{cccc} \text{a} & \text{i} & \text{w} & \text{d} \end{array} \\ \begin{pmatrix} 0.3 & 0.2 & 0.3 & 0.2 \\ 0.3 & 0.4 & 0.0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}.$$

- Changes in state occur at the end of the year.
- The only benefit is a death benefit of \$100,000 payable at the end of the year of death.
- Interest rate is $i = 5\%$.
- The insured is active and healthy at policy issue.

Calculate the actuarial present value of the benefits for this insurance.

Question No. 5:

An actuarial student uses one of two manuals (Manual X and Manual Y) to prepare for an actuarial exam. You are given the following states:

(a) uses Manual X, but fails,

(b) uses Manual X, and passes,

(c) uses Manual Y, but fails, and

(d) uses Manual Y, and passes

Assume time-homogeneous Markov Chain with annual transition probability matrix given by

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 0.3 & 0.1 & 0.5 & 0.1 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix} \end{array}.$$

The actuarial exam is given once a year. The student, on his first attempt, just failed the actuarial exam using Manual Y.

Calculate the probability that the student will pass the actuarial exam only on his fourth attempt.

Question No. 6:

A college football player may withdraw from the university for one of three possible reasons:

(1) academic failure;

(2) joins professional league; or

(3) other reasons.

According to a university's historical experience, the following table provides probabilities of withdrawals from each of these causes:

class	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
first year	0.10	0.15	0.05
second year	0.08	0.25	0.04
third year	0.05	0.30	0.03
fourth year	0.02	0.50	0.02

Calculate the probability that an entering freshman joining the university's football team will ever graduate by completing its 4-year degree program.

Question No. 7:

For a fully discrete whole life insurance policy of \$20,000 on (40) with level annual premiums, the asset shares at the end of years 10 and 11 are respectively:

$$\mathrm{AS}_{10} = 1,990.0 \quad \text{and} \quad \mathrm{AS}_{11} = 2,010.8$$

In addition, you are given:

- The contract premium is one cent for every dollar of insurance.
- The percent of premium expense in year 11 is 5%.
- The fixed expense in year 11 is \$10.
- $i = 7.0\%$
- $q_{50}^{(w)} = 0.15$ and $q_{50}^{(d)} = 0.03$

Calculate the cash value payable upon withdrawal at the end of 11 years.

Question No. 8:

For a Type A Universal Life policy with a total death benefit of $ 75,000, you are given:

policy year	annual premium deposit	percent of premium charge	annual fixed expense charge	annual cost of insurance rate per 1,000	interest credited
1	$ 1,500	10%	$ 10	2.5	5%
2	$ 1,250	5%	$ 10	3.0	5%

Calculate the account value at the end of two years.

Question No. 9:

For a Universal Life policy with death benefit equal to \$5,000 plus account value issued to (50), you are given:

- The premium paid at the beginning of each year is \$2,500.
- Expense charges in each year are 2% of premium plus \$20.
- The cost of insurance rate is equal to 120% of the mortality rate at the attained age based on the `Illustrative Life Table`.
- $i^c = 5\%$ for all years
- $i^q = 4\%$ for all years

Calculate the corridor factor at the end of the second year.

Question No. 10:

For a Type A universal life insurance policy issued to (45), you are given:

- The total death benefit, payable at the end of the year of death, is \$250,000.
- The annual premium at the beginning of the first year is \$3,000.
- COI charges are paid at the beginning of each year, and the COI rate is equal to 125% of the mortality rate at the attained age according to the `Illustrative Life Table`.
- $i^q = 0.04$ and $i^c = 0.05$ for all years.
- Expense charges consist of: (a) 3% of each annual premium, plus (b) a fixed amount of 40 at the beginning of each year.
- Account values are calculated once a year.

Calculate the decrease in the account value if this were a Type B Universal Life with an additional death benefit of \$250,000.

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000{}_5E_x$	$1000{}_{10}E_x$	$1000{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000{}_5E_x$	$1000{}_{10}E_x$	$1000{}_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110