MATH 3631

Actuarial Mathematics II

Class Test 2 - 3:35-4:50 PM

Wednesday, 20 April 2016

Time Allowed: 1 hour

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: Emil Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

An insurance company issued 500,000 fully discrete whole life insurance policies to lives all exactly age 40 on January 1, 2002. Each policy issued has a death benefit of $50,000 with an annual gross premium of $375.

Year 2012

2002 (40) → 2012 50, 2013

You are given:

- The following values in Year 2012:

	anticipated	actual
Expenses as a percent of premium	0.05	0.04
Fixed expenses	7	10
Death settlement expense	100	150
Annual effective rate of interest	0.050	0.063
q_{50}	0.0012	-

- The gross premium reserves per policy at the end of Years 2011 and 2012 are, respectively:

$$_{10}V^g = 3,539 \text{ and } _{11}V^g = 4,027$$

- A total of 495,100 remain in force at the beginning of Year 2012.
- Gains and losses are calculated in the following order: expenses then mortality then interest.
- There is a total of zero gain or loss for Year 2012.

Calculate the gain or loss due to mortality in Year 2012.

$V^E = 495100(4027)$

$$V^A = \cancel{495100} * \left[\frac{\overbrace{(3539+375(1-.04)-10)(1.063)}^{4134.007} - 50150\, q_{50}^*}{1-q_{50}^*} \right] = \cancel{495100} * 4027$$

equal since zero loss/gain

Solving for $q_{50}^* \Rightarrow q_{50}^* = \dfrac{4134.007 - 4027}{50150 - 4027} \approx 0.002320036$

Gain due to mortality:

$$(50150 - 4027)(0.0012 - 0.002320036)(495100)$$

use actual expenses

$$\approx \underbrace{-25,576,569}_{\text{big loss !!}}$$

Question No. 2:

For a fully discrete whole life insurance policy of \$150,000 issued to age 40, you are given:

- Mortality follows the `Illustrative Life Table`.
- $i = 6\%$

Calculate the Full Preliminary Term (FPT) reserve at the end of 15 years.

$$\alpha = \underbrace{150000 * \overset{1/1.06}{v} \times \overset{2.78/1000}{q_{40}}}_{393.3962}$$

$$\underbrace{P\ddot{a}_{40}}_{150000 A_{40}} = \underset{393.3962}{\alpha} + \beta \underset{13.8166}{a_{40}}$$

A_{40}: .16132

$$\Rightarrow \text{Solving for } \beta = \frac{150000(.16132) - \alpha}{13.8166} = 1722.899$$

$$_{15}V^{FPT} = 150000 \underset{.30514}{A_{55}} - \beta \underset{12.2758}{\ddot{a}_{55}} = \underline{24621.64}$$

Equivalently,

$$P = 150000 A_{41}/\ddot{a}_{41} = 150000(.16869)/14.6864 = 1722.921$$

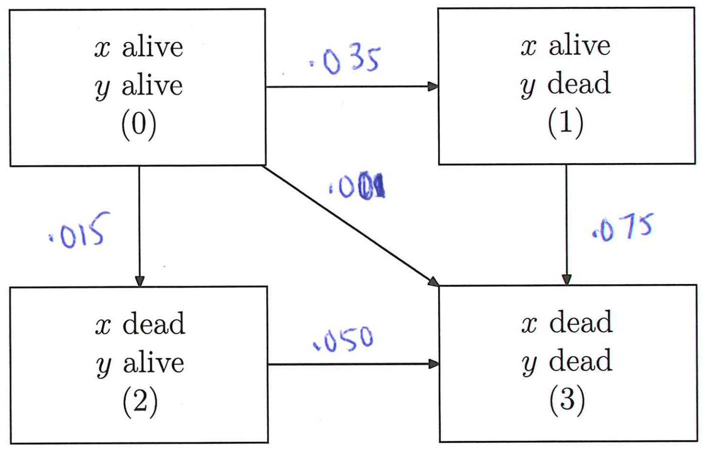
$$_{15}V^{FPT} = {}_{14}V \text{ for } (41) = 150000 A_{55} - \underset{1722.921}{P} \ddot{a}_{55}$$

$$= \underline{\underline{24,620.77}}$$

not exact because of <u>rounding</u>!

Question No. 3:

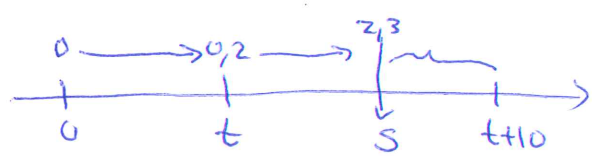
The joint lifetime of (x) and (y) is being modeled as:



Assume that at time 0, both (x) and (y) are alive. All transition intensities are constant and independent of age:

$$\mu^{01} = 0.035, \quad \mu^{02} = 0.015, \quad \mu^{03} = 0.001, \quad \mu^{13} = 0.075, \quad \text{and} \quad \mu^{23} = 0.050$$

Calculate the probability that the first death is (x) and that the death of (y) occurs within the subsequent 10 years.



$$\text{probability} = \int_0^\infty {}_tp^{\overline{00}} \mu^{02} \cdot \int_t^{t+10} {}_{s-t}p^{\overline{22}} \mu^{23} \, ds \, dt$$

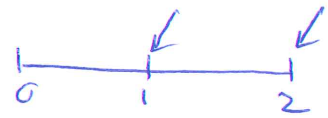
$$e^{-.051t} \cdot .015 \int_t^{t+10} e^{-.05(s-t)} \, .05 \, ds \, dt$$

$$= .015(.05) \int_0^\infty e^{-.051t} \underbrace{\int_0^{10} e^{-.05w} \, dw}_{\text{after transformation}} \, dt$$

$$\frac{1}{.05}(1-e^{-.5})$$

$$= \frac{.015(\cancel{.05})(1-e^{-.5})}{\cancel{.05}} \frac{1}{.051} = \underline{\underline{0.1157263}}$$

Question No. 4:



An insurer issues a special 2-year insurance to a high risk individual. You are given the following time-homogeneous Markov Chain model:

- The possible states are active (a), disabled (i), withdrawn (w) or dead (d) with annual transition matrix:

$$\begin{array}{c c} & \begin{array}{cccc} a & i & w & d \end{array} \\ \begin{array}{c} a \\ i \\ w \\ d \end{array} & \begin{pmatrix} 0.3 & 0.2 & 0.3 & 0.2 \\ 0.3 & 0.4 & 0.0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \end{array}$$

- Changes in state occur at the end of the year.
- The only benefit is a death benefit of \$100,000 payable at the end of the year of death.
- Interest rate is $i = 5\%$.
- The insured is active and healthy at policy issue.

Calculate the actuarial present value of the benefits for this insurance.

Possible transitions	probability	discounted cash flow
$a \to d$	0.2	$100{,}000\,v$
$a \to a \to d$	$0.3(0.2) = 0.06$ $\Big\rangle\ .12$	$100{,}000\,v^2$
$a \to i \to d$	$0.2(0.3) = 0.06$	where $v = \frac{1}{1.05}$

note that while active, possible to withdraw but cannot go anywhere else

$$APV(\text{benefits}) = 100{,}000\,v\,(.2 + .12v)$$

$$= \cancel{28{,}117.91}\ \ \underline{\underline{29{,}931.97}}$$

Question No. 5:

An actuarial student uses one of two manuals (Manual X and Manual Y) to prepare for an actuarial exam. You are given the following states:

(a) uses Manual X, but fails,

(b) uses Manual X, and passes,

(c) uses Manual Y, but fails, and

(d) uses Manual Y, and passes

Assume time-homogeneous Markov Chain with annual transition probability matrix given by

matrix

beginning state →

$$\begin{array}{cc} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{pmatrix} 0.3 & 0.1 & 0.5 & 0.1 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.5 & 0.2 & 0.2 & 0.1 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{pmatrix} \end{array} . = Q$$

The actuarial exam is given once a year. The student, on his first attempt, just failed the actuarial exam using Manual Y.

Calculate the probability that the student will pass the actuarial exam only on his fourth attempt.

initial state vector

$(0\ 0\ 1\ 0)\,Q = (.5\ \ .2\ \ .2\ \ .1)$

0 1 2 3
Fail Fail Pass

$(.5\ .2\ .2\ .1)\,Q = (.25\ \ .29\ \ .29\ \ .17)$

$(.25\ .29\ .29\ .17)\,Q = (.22\ \ .373\ \ .183\ \ .224)$

probability of passing on the 4th attempt $= (.373 + .224) =$

$- (.29 + .17) = 0.137$

Question No. 6:

A college football player may withdraw from the university for one of three possible reasons:

(1) academic failure;

(2) joins professional league; or

(3) other reasons.

According to a university's historical experience, the following table provides probabilities of withdrawals from each of these causes:

class	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$	$q_x^{(\tau)}$
first year	0.10	0.15	0.05	.30
second year	0.08	0.25	0.04	.37
third year	0.05	0.30	0.03	.38
fourth year	0.02	0.50	0.02	.54

Calculate the probability that an entering freshman joining the university's football team will ever graduate by completing its 4-year degree program.

0 1 2 3 4

no withdraw no withdraw no withdraw no withdraw

probability $= (1-.3)(1-.37)(1-.38)(1-.54)$

$= \underline{\underline{0.1257732}}$

Question No. 7:

For a fully discrete whole life insurance policy of \$20,000 on (40) with level annual premiums, the asset shares at the end of years 10 and 11 are respectively:

$$\mathrm{AS}_{10} = 1,990.0 \quad \text{and} \quad \mathrm{AS}_{11} = 2,010.8$$

In addition, you are given: 20000 * .01 = 200

- The contract premium is one cent for every dollar of insurance.
- The percent of premium expense in year 11 is 5%.
- The fixed expense in year 11 is \$10.
- $i = 7.0\%$
- $q_{50}^{(w)} = 0.15$ and $q_{50}^{(d)} = 0.03$

Calculate the cash value payable upon withdrawal at the end of 11 years.

$$\underbrace{AS_{11}}_{2010.8} = \frac{(1990 + 200(.95) - 10)(1.07) - 20000(.03) - CV_{11}(.15)}{1 - .03 - .15}$$

Solving for CV_{11}, we get

$$CV_{11} = \frac{(1990 + 200(.95) - 10)(1.07) - 20000(.03) - 2010.8(.82)}{0.15}$$

$$= \underline{\underline{486.96}}$$

Question No. 8:

Type A ADB = 75000 - AV

For a Type A Universal Life policy with a total death benefit of \$ 75,000, you are given:

policy year	annual premium deposit	percent of premium charge	annual fixed expense charge	annual cost of insurance rate per 1,000	interest credited
1	\$ 1,500	10%	\$ 10	2.5	5%
2	\$ 1,250	5%	\$ 10	3.0	5%

Calculate the account value at the end of two years.

$$AV_1 = \frac{(1500(.9)-10)(1.05) - 75000(.0025)}{1-.0025} = \underline{\underline{1222.556}}$$

$$AV_2 = \frac{(1222.556 + 1250(.95) - 10)(1.05) - 75000(.0030)}{1-.0030} = \underline{\underline{2301.965}}$$

Question No. 9:

For a Universal Life policy with death benefit equal to \$5,000 plus account value issued to (50), you are given:

- The premium paid at the beginning of each year is \$2,500.
- Expense charges in each year are 2% of premium plus \$20.
- The cost of insurance rate is equal to 120% of the mortality rate at the attained age based on the `Illustrative Life Table`.
- $i^c = 5\%$ for all years
- $i^q = 4\%$ for all years

Calculate the corridor factor at the end of the second year.

$$AV_1 = \left(2500(.98) - 20 - \frac{1.2\, \overset{.00592}{q_{50}}}{1.04} * 5000\right)(1.05) = 2515.638$$

$$AV_2 = \underbrace{\left(2515.638 + 2500(.98) - 20 - \frac{1.2\, \overset{.00642}{q_{51}}}{1.04} * 5000\right)(1.05)}$$

$$= 5154.03$$

$$cf = \frac{AV_2 + 5000}{AV_2} = \frac{5154.03 + 5000}{5154.03} = 1.970115$$

$\Downarrow$

relatively low

Question No. 10:

For a Type A universal life insurance policy issued to (45), you are given:

- The total death benefit, payable at the end of the year of death, is \$250,000.
- The annual premium at the beginning of the first year is \$3,000.
- COI charges are paid at the beginning of each year, and the COI rate is equal to 125% of the mortality rate at the attained age according to the `Illustrative Life Table`.
- $i^q = 0.04$ and $i^c = 0.05$ for all years.
- Expense charges consist of: (a) 3% of each annual premium, plus (b) a fixed amount of 40 at the beginning of each year.
- Account values are calculated once a year.

Calculate the decrease in the account value if this were a Type B Universal Life with an additional death benefit of \$250,000.

For Type A, we set

$$AV_1 = \underbrace{\frac{(3000(.97)-40)(1.05) - 250000\,(1.25\,q_{45} \times 1.05)/1.04}{1 - 1.25\,q_{45} \times 1.05/1.04}}$$

$$= 1760.367$$

If this were a Type B with ADB = 250,000 and all others stay
prems, exp, etc.

then $AV_1 = \underbrace{(3000(.97)-40)(1.05) - 250000\,(1.25\,q_{45} \times 1.05)/1.04}$

$$= 1751.481$$

$1760.367 - 1751.481 = \underline{\underline{8.886469}}$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK