MATH 3631

Actuarial Mathematics II

Class Test 1 - 3:35-4:50 PM

Wednesday, 19 February 2020

Time Allowed: 1 hour and 15 minutes

Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: EMIL **Student ID:** Suggested Solutions

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

For a fully discrete whole life insurance of 10 issued to (50), you are given:

- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- L_{10} is the prospective loss random variable at the end of year 10.

Calculate $\mathrm{E}[L_{10}]$.

$$P = \frac{10 A_{50}}{\ddot{a}_{50}} \quad A_{50} = .18931, \ \ddot{a}_{50} = 17.0245$$

$$= 0.1111986$$

$$E[L_{10}] = {}_{10}V = APV(FB_{10}) - APV(FP_{10})$$

$$= 10 A_{60} - P \ddot{a}_{60}$$

$$A_{60} = .29028$$

$$\ddot{a}_{60} = 14.9041$$

$$= 1.245485 \approx 1.25$$

or alternatively, use

$${}_{10}V = 10 * \left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{50}}\right)$$

$$= 10 * \left(1 - \frac{14.9041}{17.0245}\right)$$

$$= 1.245499 \approx \underline{\underline{1.25}}$$

Question No. 2:

For a fully discrete whole life insurance of 10 issued to (50), you are given:

- Mortality follows the `Survival Ultimate Life Table`.
- $i = 0.05$
- L_5 is the prospective loss random variable at the end of year 5.

Calculate $\mathrm{Var}(L_5)$.

$d = .05/1.05$

$B = 10$

$P = 0.1111986$

(qustion 1)

$$\mathrm{Var}(L_5) = \left[{}^2A_{55} - (A_{55})^2 \right] * \left(B + \frac{P}{d} \right)^2$$

$${}^2A_{55} = .07483$$

$$A_{55} = .23524$$

$$= \left(10 + \frac{0.1111986}{.05/1.05} \right)^2 \left[.07483 - (.23524)^2 \right]$$

$$= 2.965855 \approx \underline{\underline{2.97}}$$

Question No. 3:

For a fully discrete whole life insurance of 100 on (35), you are given:

- First year expenses are 10% of the gross premium.
- Renewal expenses are 5% of the gross premium.
- Expenses are incurred at the beginning of the policy year.
- Mortality follows the **Survival Ultimate Life Table**.
- $i = 0.05$
- Gross premium is calculated according to the equivalence principle.

$A_{35} = .09653$

$\ddot{a}_{35} = 18.9728$

Calculate the gross premium reserve at the end of year 10.

$$G\ddot{a}_{35} = 100A_{35} + .05G + .05G\ddot{a}_{35}$$

$$(.95\ddot{a}_{35} - .05)G = 100A_{35}$$

$$G = \frac{100A_{35}}{.95\ddot{a}_{35} - .05} = 0.5370487$$

$${}_{10}V^{g} = APV(FB_{10}) + APV(FE_{10}) - APV(FG_{10})$$

$$= 100A_{45} + G(.05)\ddot{a}_{45} - G\ddot{a}_{45}$$

$$= 100A_{45} - .95G\ddot{a}_{45}$$

$A_{45} = .15161$

$\ddot{a}_{45} = 17.8162$

$$= 6.071241 \approx \underline{\underline{6.07}}$$

Question No. 4:

remove gross

For a whole life insurance on (40), you are given:

- The death benefit is 1000, payable at the end of the year of death.
- There is only a single ~~gross~~ premium of 125, payable at policy issue.
- There are no expenses.
- Mortality follows the `Survival Ultimate Life Table`.
- $\delta = 0.05$
- L_{10} is the loss for this policy in year 10.

Calculate $\Pr[L_{10} > \cancel{200}\ 180]$.

$L_{10} = PVFB_{10} - \cancel{PVFP_{10}}^{\,0}$

no future premium at time 10

$= 1000\, v^{K+1}$

$K = K_{50}$ curtate lifetime of (40) after 10 years

$\Pr[L_{10} > 180] = \Pr[1000\, v^{K+1} > 180]$

$= \Pr\left[(K+1)\log v > \log\left(\frac{180}{1000} = 0.18\right)\right]$

$= \Pr\left[K < \underbrace{\frac{\log .18}{-\delta} - 1}_{33.29597}\right] = \Pr[K \leq 33]$

$= {}_{34}q_{50} = 1 - \frac{l_{84}}{l_{50}} = \frac{64506.5}{98576.4}$ $= 0.3456192$

$\approx \underline{\underline{0.35}}$

$65+24=89$

Question No. 5:

For a fully discrete whole life insurance of 1000 on (65), you are given:

- The net premium reserve at the end of year 24 is 502.58.
- $q_{89} = \cancel{0.22}$ 0.17
- $i = 0.05$
- $A_{65} = 0.6135$

Calculate ${}_{25}V$, the net premium reserve at the end of year 25.

$P = 1000 \dfrac{A_{65}}{\ddot{a}_{65}} = 75.58677$

$A_{65} = 0.6135$

$\ddot{a}_{65} = \dfrac{1-A_{65}}{d} = \dfrac{1-.6135}{.05/1.05} = 8.1165$

${}_{24}V = 502.58$

$${}_{25}V = \frac{({}_{24}V + P)(1.05) - 1000\, q_{89}}{1 - q_{89}}$$

$q_{89} = .17$

${}_{25}V = 527.85$

$\cancel{527.85}$ 526.5965 $= (502.58 + P)(1.05) - (1000 - 527.85)\, q_{89}$

.1677965

$q_{89} = 0.17$

$q_{89} =$

${}_{25}V = 526.5965$

≈ 527

Question No. 6:

For a special fully discrete whole life insurance on (40), you are given:

- The death benefit is 20 in the first year and 10 in all subsequent years.
- $q_{40} = 0.0020 \qquad q_{50} = 0.0025$
- $i = 0.03$
- $A_{40} = 0.37 \qquad A_{50} = 0.48$
- Deaths within one year are uniformly distributed throughout the year.

Calculate ${}_{10.4}V$, the net premium reserve in year 10.4.

$A_{40} = .37$

$\ddot{a}_{40} = \dfrac{1 - A_{40}}{d} = \dfrac{1-.37}{.03/1.03} = 21.63$

need ${}_{10}V = ?$

$P\ddot{a}_{40} = \underbrace{10 \cdot v\, q_{40}}_{\text{extra 10 death benefit in 1st yr}} + 10A_{40}$

$P = \underbrace{\left(10\, v q_{40} + 10 A_{40}\right) / \ddot{a}_{40}} = .1719564$

$${}_{10}V = 10A_{50} - P\ddot{a}_{50}$$
$$10(.48) - .1719564\left(\frac{1-A_{50}}{d}\right) = 1.730005 \approx 1.73$$

$A_{50} = .48$

$\ddot{a}_{50} = \underbrace{\dfrac{1-A_{50}}{d}}$
17.85333

Thus,

$${}_{10.4}V = \frac{({}_{10}V + P)(1.03)^{.4} - 10 * .4\, q_{50}\, v^{0.60}}{1 - 0.4 * q_{50}}$$

$= \underline{1.916517}$

Question No. 7:

For a 3-year term insurance on (60), you are given:

- There is only one single premium, P, payable at issue.
- The death benefit, payable at the end of the year of death, is equal to 10 plus the benefit reserve.
- $q_{60+k} = 0.01$, for $k = 0, 1, 2, \ldots$
- $i = 0.04$

Calculate P.

$$_0V = 0$$

$$_1V = (0 + P)(1.04) - (10 + \cancel{_1V} - \cancel{_1V})(.01)$$

$$= P(1.04) - 10(.01)$$

$$_2V = (P(1.04) - 10(.01))(1.04) - (10 + \cancel{_2V} - \cancel{_2V})(.01)$$

$$= P(1.04)^2 - 10(.01)[(1.04) + 1]$$

$$\vdots$$

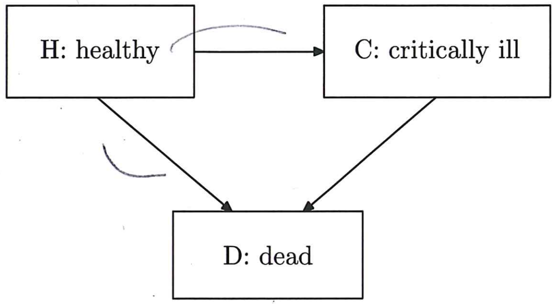
$$_3V = P(1.04)^3 - 10(.01)[1.04^2 + 1.04 + 1] = 0$$ term

$$P = \frac{10(.01)(1.04^2 + 1.04 + 1)}{1.04^3}$$

$$= 0.2775091 \approx 0.28$$

Question No. 8:

You are given the following critical illness multiple state model:



You are given that all the forces of transition are independent of age and time with:

$$\mu^{\text{HC}} = 0.025 \qquad \mu^{\cancel{\text{HC}}\ \text{HD}} = 0.018 \qquad \mu^{\text{CD}} = 0.048$$

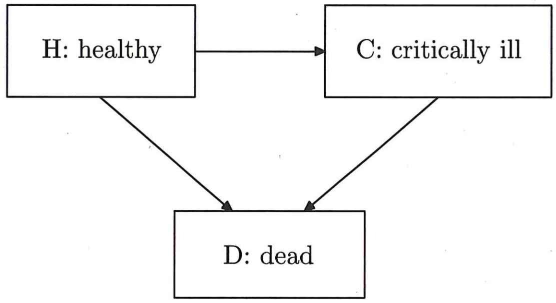
Calculate the probability that a Healthy policyholder will remain healthy at the end of 10 years.

$$_{10}p^{HH} = e^{-\int_0^{10} (\mu^{HC} + \mu^{HD})\,dt}$$

$$_{10}p^{\overline{HH}} = e^{-10(.025+.018)} = \underline{\underline{0.6505091}}$$

Question No. 9:

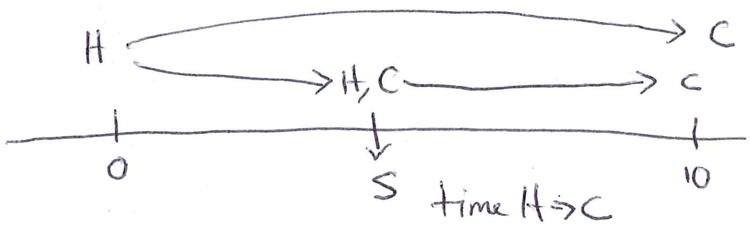
You are given the following critical illness multiple state model:



You are given that all the forces of transition are independent of age and time with:

$$\mu^{HC} = 0.025 \qquad \mu^{\cancel{HC}\ HD} = 0.018 \qquad \mu^{CD} = 0.048$$

Calculate the probability that a Healthy policyholder will be critically ill at the end of 10 years.



$$_{10}p^{HC} = \int_0^{10} e^{-.043s} \cdot .025 \; \cancel{.018} \; e^{-.048(10-s)} \, ds$$

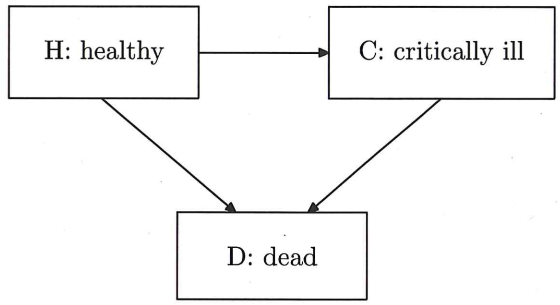
$$= .025 \; \cancel{.018} \; e^{-.48} \int_0^{10} e^{+.005s} \, ds$$

$$= \underbrace{\frac{.025 \; \cancel{.018}}{.005} e^{-.48} \left(e^{+.05} - 1 \right)}$$

~~[illegible]~~ 0.1586285

Question No. 10:

You are given the following critical illness multiple state model:



You are given that all the forces of transition are independent of age and time with:

$$\mu^{\mathrm{HC}} = 0.025 \qquad \mu^{\cancel{\mathrm{HC}}\ \mathrm{HD}} = 0.018 \qquad \mu^{\mathrm{CD}} = 0.048$$

Calculate the probability that a Critically Ill policyholder will remain critically ill at the end of 10 years.

$${}_{10}p^{\overline{CC}} = {}_{10}p^{CC} = e^{-.048(10)} = 0.6187834$$

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK