MATH 3631
Actuarial Mathematics II
Class Test 1 - 3:35-4:50 PM
Wednesday, 20 February 2019
Time Allowed: 1 hour and 15 minutes
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: _____ **Student ID:** _____

- **There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.**
- **Please provide details of your workings in the appropriate spaces provided; partial points will be granted.**
- **Please write legibly.**
- **Anyone caught writing after time has expired will be given a mark of zero.**

Question No. 1:

For a fully discrete whole life insurance of 1000 on (35), you are given:

- First year expenses are 20% of the gross premium.
- Renewal expenses are 5% of the gross premium.
- Expenses are incurred at the beginning of the policy year.
- Gross premium is calculated according to the equivalence principle.
- Mortality follows the `Survival Ultimate Life Table` with $i = 0.05$.

Calculate the gross premium reserve at the end of year 20.

Question No. 2:

Your company issues fully discrete whole life policies to a group of lives age 40. For each policy, the death benefit is 100 and you are given:

- Assumed mortality and interest are the `Survival Ultimate Life Table` at 5%.
- Assumed expenses are 5% of gross premium, payable at the beginning of each year, and 5 to process each death claim, payable at the end of the year of death.
- Annual gross premium equals 0.72.
- The gross premium reserves are ${}_{15}V = 13.64$ and ${}_{16}V = 14.87$.

During year 16, actual experience is as follows:

- There are 20,000 lives in force at beginning of the year with 45 deaths during the year.
- Investment earnings equal 5%.
- Expenses are 6% of gross premium and 1 to process each death claim.

Gains or losses are calculated according to: interest $\rightarrow$ mortality $\rightarrow$ expenses.

Calculate the gain or loss due to expenses during year 16.

Question No. 3:

For a whole life insurance on (40), you are given:

- The death benefit is 1000, payable at the end of the year of death.
- There is only a single gross premium of 125, payable at policy issue.
- There are no expenses.
- Mortality follows the `Survival Ultimate Life Table`.
- $\delta = 0.05$
- L_{10} is the loss for this policy in year 10.

Calculate $\Pr[L_{10} > 200]$.

Question No. 4:

For a special fully discrete whole life insurance on (40), you are given:

- The death benefit is 20 in the first year and 10 in all subsequent years.
- $q_{40} = 0.0020 \qquad q_{50} = 0.0025$
- $i = 0.03$
- $A_{40} = 0.37 \qquad A_{50} = 0.48$
- Deaths within one year are uniformly distributed throughout the year.

Calculate ${}_{10.5}V$, the net premium reserve in year 10.5.

Question No. 5:

For a 3-year endowment insurance on (62), you are given:

- The death benefit, payable at the end of the year of death, is equal to 10 plus the benefit reserve.
- The endowment benefit is 40.
- Level premiums, P, are payable annually at the beginning of each year.
- $q_{62+k} = 0.025$, for $k = 0, 1, 2, \ldots$
- $i = 0.05$

Calculate P.

Question No. 6:

For a fully discrete whole life insurance of 5 on (60), you are given:

- $q_{60} = 0.003 \qquad q_{61} = 0.004$
- $i = 0.05$
- $A_{60} = 0.30$
- ${}^2A_{60} = 0.10$
- L_1 is the insurer's prospective loss at time 1 for this policy.

Calculate $\text{Var}(L_1)$.

Question No. 7:

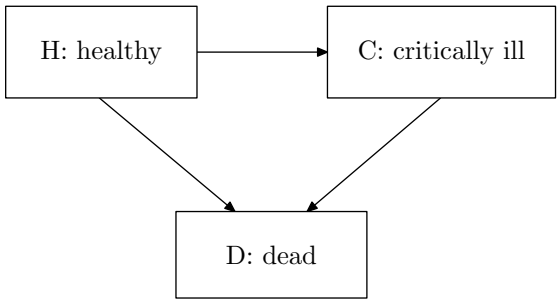
For a fully discrete whole life insurance of 100 on (x), you are given:

- Expenses, incurred at the beginning of each year, equal 10% of the gross premium in the first year and 5% of the gross premium in subsequent years.
- Both net and gross premiums are calculated using the equivalence principle.
- $i = 0.04$
- $\ddot{a}_x = 12.5$
- $\ddot{a}_{x+10} = 9.4$

Calculate ${}_{10}V^e$, the expense reserve (or DAC) at the end of year 10.

Question No. 8:

A life insurer uses the following three-state model to price critical illness policies issued to healthy policyholders at time $t = 0$:



You are given:

- All forces of transition are constant, that is, independent of age and time with:

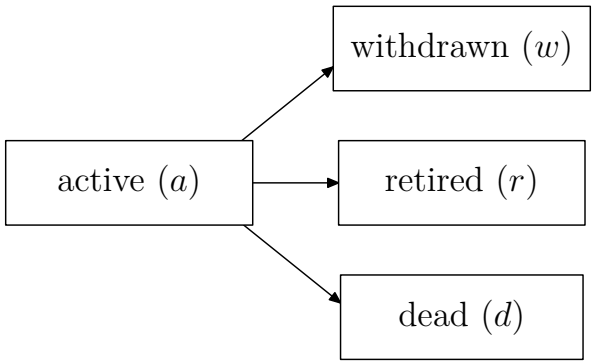
  $$\mu^{HD} = 0.017 \qquad \mu^{CD} = 0.055$$

- The probability that a healthy policyholder will be healthy at the end of 10 years is 0.64.
- μ^{HC} is the force of transition from state H to C.

Calculate μ^{HC}.

Question No. 9:

You are given the following retirement model:



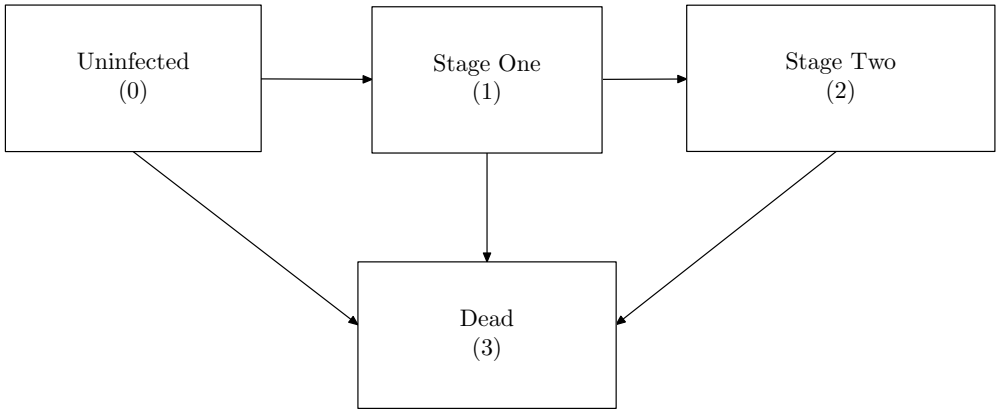
You are given:

- All forces of transition are constant, that is, independent of age and time.
- μ^{aw}, μ^{ar} and μ^{ad} denote the forces of transitions from being Active to the other states, respectively.
- $\mu^{ad} = 0.010$
- The probability that you Withdraw, given you leave the Active state within 10 years, is 0.250.
- The probability that you Die, given you leave the Active state within 10 years, is 0.625.

Calculate μ^{ar}.

Question No. 10:

A disease progresses according to the following multiple state model:



All transition intensities are constant and independent of age:

$$\mu^{01} = 0.005, \quad \mu^{12} = 0.08, \quad \mu^{03} = 0.01, \quad \mu^{13} = 0.05, \quad \text{and} \quad \mu^{23} = 0.40$$

Calculate the probability that an Uninfected person today will reach Stage Two at the end of 10 years.

Bonus questions: 1 point each

State the first and last name of your Math 3631 instructor: _____

State one reason why you would buy life insurance (now at your age while still in college - assume you can afford): _____

State one reason why insurers must hold reserves: _____

State one reason why insurers will be reluctant to issue you a policy, with the first premium payable a year from issue: _____

State one reason why gross premium reserves are generally lower than net premium reserves (for the same type of coverage): _____

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK