MATH 3631
Actuarial Mathematics II
Class Test 1 - 3:35-4:50 PM
Wednesday, 20 February 2019
Time Allowed: 1 hour and 15 minutes
Total Marks: 100 points

Please write your name and student number at the spaces provided:

Name: Emil   Student ID: Suggested Solutions

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

Question No. 1:

For a fully discrete whole life insurance of 1000 on (35), you are given:

- First year expenses are 20% of the gross premium.
- Renewal expenses are 5% of the gross premium.
- Expenses are incurred at the beginning of the policy year.
- Gross premium is calculated according to the equivalence principle.
- Mortality follows the `Survival Ultimate Life Table` with $i = 0.05$.

Calculate the gross premium reserve at the end of year 20.

Let G be the gross premium

$$APV(FP_0) = APV(FB_0) + APV(FE_0)$$

$$G\ddot{a}_{35} = 1000A_{35} + .15G + .05G\ddot{a}_{35}$$

$$G = \frac{1000\overbrace{A_{35}}^{.09653}}{.95\underbrace{\ddot{a}_{35}}_{18.9728} - .15} = 5.400534$$

$${}_{20}V^{g} = APV(FB_{20}) + APV(FE_{20}) - APV(FP_{20})$$

$$= 1000A_{55} + .05G\ddot{a}_{55} - G\ddot{a}_{55}$$

$$= 1000\underbrace{A_{55}}_{.23524} - .95G\overbrace{\ddot{a}_{55}}^{16.0599}$$

$$= 152.8446$$

Question No. 2:

Your company issues fully discrete whole life policies to a group of lives age 40. For each policy, the death benefit is 100 and you are given:

- Assumed mortality and interest are the `Survival Ultimate Life Table` at 5%.
- Assumed expenses are 5% of gross premium, payable at the beginning of each year, and 5 to process each death claim, payable at the end of the year of death.
- Annual gross premium equals 0.72.
- The gross premium reserves are ${}_{15}V = 13.64$ and ${}_{16}V = 14.87$.

During year 16, actual experience is as follows:

- There are 20,000 lives in force at beginning of the year with 45 deaths during the year.
- Investment earnings equal 5%.
- Expenses are 6% of gross premium and 1 to process each death claim.

Gains or losses are calculated according to: interest → mortality → expenses.

Calculate the gain or loss due to expenses during year 16.

Interest: 0

Mortality: not needed

use actual since mortality already accounted

Expenses: $$\underbrace{20000\left[(\underset{\text{expected}}{.05G}-\underset{\text{actual}}{.06G})(1.05)+(\underset{\text{expected}}{5}-\underset{\text{actual}}{1})*\frac{45}{20000}\right]}_{+28.8}$$

expense at death

+28.8 a gain, despite beginning expenses were higher than actual

Question No. 3:

For a whole life insurance on (40), you are given:

- The death benefit is 1000, payable at the end of the year of death.
- There is only a single gross premium of 125, payable at policy issue.
- There are no expenses.
- Mortality follows the `Survival Ultimate Life Table`.
- $\delta = 0.05$
- L_{10} is the loss for this policy in year 10.

Calculate $\Pr[L_{10} > 200]$.

Since no more premiums to be paid, $L_{10} = PVFB_{10}$

$= 1000\, v^{K+1}$

$$= \Pr[1000\, v^{K+1} > 200]$$

$v = e^{-\delta}$

$\log v = -\delta$

$$= \Pr[(K+1)\log v > \log(0.20)]$$

$$= \Pr\Big[K < \underbrace{\frac{\log(0.20)}{-\delta} - 1}_{31.18876}\Big]$$

$\Pr[K \leq k] = {}_{k+1}q_x$

$$= \Pr[K \leq 31] = {}_{32}q_{\cancel{40}\,50} = 1 - \frac{l_{\cancel{72}\,82}}{l_{\cancel{40}\,50}}$$

$$= 1 - \frac{\cancel{89082.1}\ 70507.2}{\cancel{99338.3}\ 98576.4} = \cancel{0.1032452}\ \underline{\underline{0.2847456}}$$

Question No. 4:

For a special fully discrete whole life insurance on (40), you are given:

- The death benefit is 20 in the first year and 10 in all subsequent years.
- $q_{40} = 0.0020$ $q_{50} = 0.0025$
- $i = 0.03$
- $A_{40} = 0.37$ $A_{50} = 0.48$
- Deaths within one year are uniformly distributed throughout the year.

Calculate ${}_{10.5}V$, the net premium reserve in year 10.5.

$$P\ddot{a}_{40} = \underbrace{10vq_{40}}_{\text{extra 10 benefit in 1st yr}} + 10A_{40}$$

$$A_{40} = 0.37 \qquad \ddot{a}_{40} = \underbrace{\frac{1-A_{40}}{d}}_{21.63}$$

$$P = \frac{10vq_{40} + 10A_{40}}{\ddot{a}_{40}} = 0.1719564$$

$${}_{10}V = 10A_{50} - P\ddot{a}_{50}$$

$$= 1.730005 \approx 1.73$$

$$A_{50} = 0.48 \qquad \ddot{a}_{50} = \underbrace{\frac{1-A_{50}}{d}}_{17.85333}$$

$${}_{10.5}V = \frac{({}_{10}V + P)(1.03)^{0.5} - 10 * 0.5q_{50}v^{0.5}}{1 - 0.5 * q_{50}}$$

$$= \underline{\underline{1.939037}}$$

Question No. 5:

For a 3-year term insurance on (62), you are given:

- The death benefit, payable at the end of the year of death, is equal to 10 plus the benefit reserve.
- The endowment benefit is 40.
- Level premiums, P, are payable annually at the beginning of each year.
- $q_{62+k} = 0.025$, for $k = 0, 1, 2, \ldots$
- $i = 0.05$

Calculate P.

$$_0V = 0$$

$$_1V = (0+P)(1.05) - (10 + \cancel{_1V} - \cancel{_1V})(.025)$$

$$= P(1.05) - 10(.025)$$

$$_2V = (_1V + P)(1.05) - (10 + \cancel{_2V} - \cancel{_2V})(.025)$$

$$= P[1.05^2 + 1.05] - 10(.025)[1.05 + 1]$$

$$\vdots$$

$$_3V = \underbrace{P(1.05^3 + 1.05^2 + 1.05) - 10(.025)(1.05^2 + 1.05 + 1)}_{\text{equal to endowment of } 40}$$

$$P = \frac{40 + 10(.025)(1.05^2 + 1.05 + 1)}{1.05^3 + 1.05^2 + 1.05}$$

$$= 12.32223 \approx \underline{\underline{12.3}}$$

Question No. 6:

For a fully discrete whole life insurance of 5 on (60), you are given:

- $q_{60} = 0.003$ $\quad q_{61} = 0.004$
- $i = 0.05$
- $A_{60} = 0.30$
- ${}^2A_{60} = 0.10$
- L_1 is the insurer's prospective loss at time 1 for this policy.

Calculate $\text{Var}(L_1)$.

$A_x = vq_x + vp_x A_{x+1}$

or

$A_{x+1} = \dfrac{A_x - vq_x}{vp_x}$

$i = .05$

$A_{60} = 0.30$

$\ddot{a}_{60} = \underbrace{\dfrac{1-A_{60}}{d}}_{1.47}$

Let P be the premium per \$1 of benefit.

$$P = \frac{A_{60}}{\ddot{a}_{60}} = .02040816$$

Now use recursive formula

$$A_{61} = \frac{A_{60}(1.05) - q_{60}}{1-q_{60}} = 0.3129388164$$

$i @ 2\delta$

$e^{\delta} = 1+i$

$e^{2\delta} = (1+i)^2$

and

$${}^2A_{61} = \frac{{}^2A_{60}(1.05)^2 - q_{60}}{1-q_{60}} = 0.177572718$$

$$\text{Var}(L_1) = 5^2 * \left(1+\frac{P}{d}\right)^2 \left({}^2A_{61} - A_{61}^2\right)$$

$$= 0.4919395568$$

Question No. 7:

For a fully discrete whole life insurance of 100 on (x), you are given:

- Expenses, incurred at the beginning of each year, equal 10% of the gross premium in the first year and 5% of the gross premium in subsequent years.
- Both net and gross premiums are calculated using the equivalence principle.
- $i = 0.04$
- $\ddot{a}_x = 12.5$
- $\ddot{a}_{x+10} = 9.4$

Calculate ${}_{10}V^e$, the expense reserve (or DAC) at the end of year 10.

$\ddot{a}_x = 12.5$

$A_x = 1 - d\ddot{a}_x$

$P = 100\dfrac{A_x}{\ddot{a}_x} = 4.153846$

$G = \dfrac{100A_x}{.95\ddot{a}_x - .05} = 4.390958$

El = expense loading = $G - P$

$= .2371117$

${}_{10}V^g = 100A_{x+10} + .05G\ddot{a}_{x+10} - G\ddot{a}_{x+10}$

$= \cancel{100A_{x+10}} - .95G\ddot{a}_{x+10}$

$-)\quad {}_{10}V^n = \cancel{100A_{x+10}} - P\ddot{a}_{x+10}$

9.4

${}_{10}V^g - {}_{10}V^n = {}_{10}V^e = (P - .95G)\ddot{a}_{x+10}$

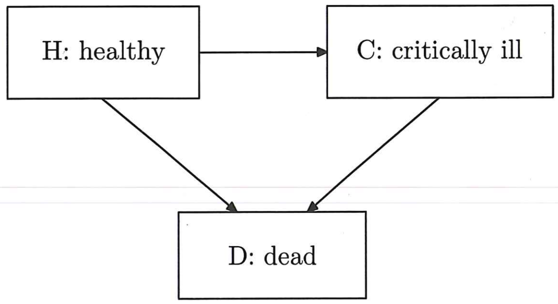
$= 0.1651$

Show this is also equal to $PV(FE_{10}) - PV(FEl_{10})$

$.05G\ddot{a}_{x+10} - (G-P)\ddot{a}_{x+10}$ ✓

Question No. 8:

A life insurer uses the following three-state model to price critical illness policies issued to healthy policyholders at time $t = 0$:



You are given:

- All forces of transition are constant, that is, independent of age and time with:

$$\mu^{HD} = 0.017 \qquad \mu^{CD} = 0.055$$

- The probability that a healthy policyholder will be healthy at the end of 10 years is 0.64.
- μ^{HC} is the force of transition from state H to C.

Calculate μ^{HC}.

$$ {}_{10}p^{HH} = {}_{10}p^{\overline{HH}} = e^{-\int_0^{10} (\mu^{HC} + \mu^{HD})\,dt}$$

$$= e^{-\mu^{HC} \cdot 10} \times e^{-.017(10)} = 0.64$$

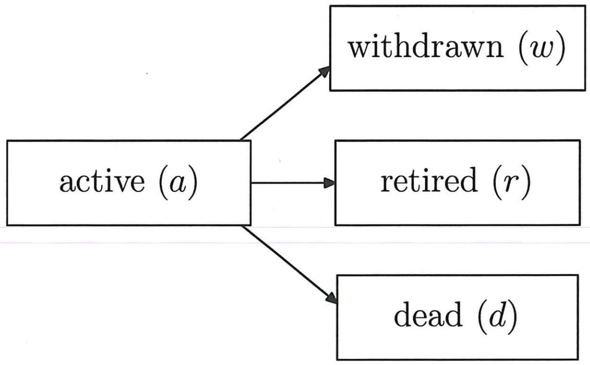
$$e^{-10\mu^{HC}} = 0.64 \times e^{0.17}$$

$$-10\mu^{HC} = \log(0.64 \times e^{0.17})$$

$$\mu^{HC} = -\frac{1}{10}\log(0.64 \times e^{0.17}) = 0.02762871$$

Question No. 9:

You are given the following retirement model:



You are given:

- All forces of transition are constant, that is, independent of age and time.
- μ^{aw}, μ^{ar} and μ^{ad} denote the forces of transitions from being Active to the other states, respectively.
- $\mu^{ad} = 0.010$
- The probability that you Withdraw, given you leave the Active state within 10 years, is 0.250.
- The probability that you Die, given you leave the Active state within 10 years, is 0.625.

Calculate μ^{ar}.

$$\frac{\mu^{aw}}{\mu^{aw}+\mu^{ar}+\mu^{ad}} = 0.250$$

$$\frac{\mu^{ad}}{\mu^{aw}+\mu^{ar}+\mu^{ad}} = 0.625$$

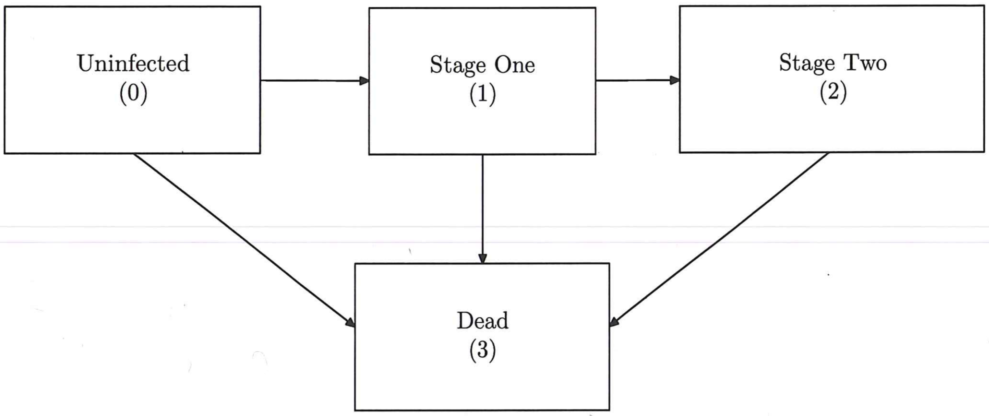
$$\Rightarrow \frac{\mu^{aw}}{\mu^{ad}} = \frac{0.250}{0.625} \Rightarrow \mu^{aw} = \frac{0.250}{0.625}(.010) = .004$$

$$\Rightarrow \frac{\mu^{ar}}{\mu^{aw}+\mu^{ar}+\mu^{ad}} = 1-0.25-0.625 = 0.125 \Rightarrow \frac{\mu^{ar}}{\mu^{ad}} = \frac{0.125}{0.625}$$

$$\Rightarrow \mu^{ar} = \frac{0.125}{0.625}\underbrace{\mu^{ad}}_{.010} = \underline{\underline{.002}}$$

Question No. 10:

A disease progresses according to the following multiple state model:



All transition intensities are constant and independent of age:

$$\mu^{01} = 0.005, \quad \mu^{12} = 0.08, \quad \mu^{03} = 0.01, \quad \mu^{13} = 0.05, \quad \text{and} \quad \mu^{23} = 0.40$$

Calculate the probability that an Uninfected person today will reach Stage Two at the end of 10 years.

$$0 \longrightarrow 0,1 \longrightarrow 1,2 \longrightarrow 2$$

0 s t 10

$$ {}_{10}p^{02} = \int_0^{10}\int_0^{t} e^{-.015s}(.005)\, e^{-.130(t-s)}(.08)\, e^{-.40(10-t)}\, ds\, dt$$

$$=(.005)(.08)e^{-4}\int_0^{10}\int_0^{t} e^{.115s}\, e^{.27t}\, ds\, dt$$

$$= .005(.08)e^{-4}\int_0^{10} \frac{1}{.115}\left(e^{.115t}-1\right) e^{.27t}\, dt$$

$$= \frac{.005(.08)}{.115}\, e^{-4}\, \underbrace{\int_0^{10}\left(e^{.385t} - e^{.27t}\right) dt}_{\left[\frac{1}{.385}\left(e^{3.85}-1\right) - \frac{1}{.25}\left(e^{2.7}-1\right)\right]} = \underline{\underline{0.00433562}}$$

Bonus questions: 1 point each

State the first and last name of your Math 3631 instructor: Emil Valdez

State one reason why you would buy life insurance (now at your age while still in college - assume you can afford): less expensive when bought at young age

State one reason why insurers must hold reserves: need money to pay promised obligations

State one reason why insurers will be reluctant to issue you a policy, with the first premium payable a year from issue: avoid having free coverage for one year

State one reason why gross premium reserves are generally lower than net premium reserves (for the same type of coverage): need to recoup large first year or acquisition expenses over policy duration

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK