

**MATH 3631**  
**Actuarial Mathematics II**  
**Class Test 1 - 5:00-6:15 PM**  
**Wednesday, 14 February 2018**  
**Time Allowed: 1 hour**  
**Total Marks: 100 points**

Please write your name and student number at the spaces provided:

Name: \_\_\_\_\_ Student ID: \_\_\_\_\_

- There are ten (10) written-answer questions here and you are to answer all ten. Each question is worth 10 points.
- Please provide details of your workings in the appropriate spaces provided; partial points will be granted.
- Please write legibly.
- Anyone caught writing after time has expired will be given a mark of zero.

**Question No. 1:**

An insurer sells a portfolio of 702 fully discrete whole life insurance policies with death benefit of 1000 to lives with independent future lifetimes, each with age  $x$ . You are given:

- The annual contract premium is 16 per policy.
- $i = 0.04$       $A_x = 0.306$       ${}^2A_x = 0.113$
- A table of  $\alpha$ -th percentile,  $z_\alpha$ , from the standard normal distribution:

$\alpha$	0.90	0.95	0.97	0.99
$z_\alpha$	1.282	1.645	1.881	2.326

Calculate the probability of a gain from this portfolio of policies.

**Question No. 2:**

For a special whole life insurance on  $(45)$ , you are given:

- Death benefit, payable at the end of the year of death, consists of 250 plus the return of all premiums without interest.
- Annual net premium of 5 is payable at the beginning of each year.
- $A_{45} = 0.25$       $\ddot{a}_{45} = 21.7$

Calculate  $(IA)_{45}$ .

**Question No. 3:**

For a whole life insurance issued to  $(40)$ , you are given:

- The death benefit is 100, payable at the end of the year of death.
- There is only a single gross premium of 14, payable at policy issue.
- Initial expenses are 4% of the single gross premium.
- Additional expenses of 0.05 incurred at the beginning of each year, including the first policy year.
- Mortality follows the Illustrative Life Table.
- $\delta = 0.05$
- $L_0$  is the loss at issue for this policy.

Calculate  $\Pr[L_0 > 15]$ .

**Question No. 4:**

For a fully discrete whole life insurance of 100 on  $(40)$ , you are given:

- First year expenses are 25% of the gross premium.
- Renewal expenses are 5% of the gross premium.
- Expenses are incurred at the beginning of the policy year.
- Gross premium is calculated according to the equivalence principle.
- Mortality follows the **Illustrative Life Table** with  $i = 0.06$ .

Calculate the gross premium reserve at the end of the second year.

**Question No. 5:**

For a fully discrete 10-year term insurance policy of 10 on  $(50)$ , you are given:

- Mortality follows the Illustrative Life Table.
- $i = 0.06$

Calculate the net premium reserve at the end of 9 years.

**Question No. 6:**

For a fully discrete whole life insurance of 1 on  $(60)$ , you are given:

- $q_{60} = 0.003$      $q_{61} = 0.004$
- $i = 0.05$
- $A_{60} = 0.30$
- ${}^2A_{60} = 0.10$
- $L_t$  is the insurer's prospective loss at time  $t$  for this policy.

Calculate  $\text{Var}(L_2)$ .

**Question No. 7:**

For a fully discrete whole life insurance of 1,000,000 on (45), you are given:

- First year expenses are 20% of the gross premium plus 3000.
- Renewal expenses are 2% of the gross premium plus 300.
- All expenses are incurred at the beginning of the policy year.
- Gross premiums are calculated using the equivalence principle.
- Mortality follows the **Illustrative Life Table** with  $i = 0.06$ .

Calculate the gross premium reserve at the end of the first policy year.



**Question No. 8:**

For a life insurance policy issued to  $(x)$ , you are given:

- Death benefit of 25,000 is payable at the end of the year of death.
- The annual net premium in year 16, payable at the beginning of the year, is 654.57.
- Deaths are assumed to be uniformly distributed over integral ages.
- $i = 0.05$      ${}_{15}V = 9,227.79$      ${}_{15.5}V = 9,889.50$

Calculate  $q_{x+15}$ .

**Question No. 9:**

For a 5-year endowment insurance on  $(60)$ , you are given:

- The death benefit, payable at the end of the year of death, is equal to 1000 plus the benefit reserve.
- The endowment benefit is 4000.
- Level premiums,  $P$ , are payable annually at the beginning of each year.
- $q_{60+k} = 0.02$ , for  $k = 0, 1, 2, \dots$
- $i = 0.05$

Calculate  $P$ .

**Question No. 10:**

For a fully discrete whole life insurance policy of 1 on  $(35)$ , you are given:

- Mortality follows the Illustrative Life Table.
- $i = 0.06$
- Expenses consist of 5% of annual gross premium, payable at the beginning of each year.
- Both the annual net premium and the annual gross premium are determined according to the equivalence principle.
- ${}_tV^n$  and  ${}_tV^g$  denote the net and gross premium reserves at time  $t$ , respectively.

Calculate  ${}_{10}V^n - {}_{10}V^g$ .

EXTRA PAGE FOR ADDITIONAL OR SCRATCH WORK